Spring 04 | Name: ______________________
Math 3770 | Final Exam | Bonetto

**The test consist of 5 exercises witha total of 26 questions. You should solve 20 of them, of your choice, to get a full score.**

1) Let $X$ and $Y$ be two continuous r.v. with joint p.d.f. given by

$$f(x,y) = \begin{cases} \lambda\mu\, e^{-(\lambda x + \mu y)} & \text{if } x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

a) compute the marginal p.d.f. of $X$ and $Y$.
b) are $X$ and $Y$ independent?

2) Let $X$ and $Y$ be two discrete r.v. with joint p.d.f $p(x, y) = P(X = x \text{ and } Y = y)$ given by

$$p(0,0) = p(0,1) = p(1,0) = p(1,1) = 0.125$$
$$p(2,2) = p(2,3) = p(3,2) = p(3,3) = 0.125$$

while $p(x, y) = 0$ in all the other cases.

a) compute the marginal p.d.f. of $X$ and of $Y$.
b) compute $E(X)$ and $E(Y)$.
c) compute $p_{X|Y}(2|0)$ and $p_{X|Y}(2|3)$.
d) are $X$ and $Y$ independent?

3) In a factory there are 2 machines that produce the same TV set (machine A and machine B). You know that machine A has a probability $p = 0.1$ to produce a defective TV set and produces 10000 TV sets every month while machine B has a probability $q = 0.05$ to produce a defective TV set and produces 2000 TV sets every month.
At the end of the first month all the 12000 TV sets produced are collected and shipped.

a) compute the probability that a randomly choosen TV set among the 12000 shipped is defective.
b) You randomly choose a TV set among the 12000 shipped and you find out that it is defective. What is the probability that it was produced by machine A?

Let now $S_A$ be the number of defective TV sets produced by machine A and $S_B$ the number of defective TV sets produced by machine B.

c) write an approximate p.d.f. for $S_A$, $S_B$ and $S_A - S_B$. (**Hint** Let $X_i$ is a r.v. equal to 1 if the $i$-th TV set produced by machine A is defective and 0 otherwise. Write $S_A$ in term of the $X_i$ and use the CLT. Similarly for $S_B$.)
d) compute $P(S_A > 2000)$ and $P(S_A < 100)$.
e) compute $P(S_A > S_B)$.

4) Let $X$ be the number of jobs a server can complete in an hour. You know that $X$ has a Poisson distribution with parameter $\lambda = 5$, *i.e.* $P(X = x) = \frac{5^x}{x!}e^{-5}$.

a) Compute the expected value of $X$.
b) Compute $P(X < 3)$.

A new server is under consideration. Observing its perfomances for 50 hours you obtain a sample of size 50 for the number of jobs the server can complete in an hour. Let $Y_1, Y_2, \dots, Y_{50}$ describe this sample. Assume that in this case the number of jobs completed has a Poisson distribution with parameter $\mu$ unknown, *i.e.* $P(Y_i = y) = \frac{\mu^y}{y!}e^{-\mu}$.

c) write the joint p.d.f. for the sample.
d) find the MLE $\hat{\theta}$ for $\mu$.
e) is $\hat{\theta}$ unbaised?

After running a real sample you obtain the following numbers

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 |
| 6 | 16 | 12 | 9 | 5 | 2 |

where the first line represent the value of $Y$ and the second the number of time you saw that value, *i.e.* you saw 0 six times, 1 sixteen times, etc..

f) compute the numerical value of $\hat{\theta}$ on the above numbers.

To have a better precision you run a larger sample of size $N = 1000$ and find a sample mean $\bar{y} = 1.97$ and variance $s^2 = 2.10$.

h) write a 95% CI for $\mu$ based on the above $\bar{y}$ and $s^2$ ($z_{\frac{\alpha}{2}} = 1.96$ for $\alpha = 0.05$).

You want to know whether the new server is better or not than the initial one.

i) which null hypothesis $H_0$ would you choose?
j) which alternative hypothesis $H_a$ would you choose?
k) at a confidence level of 95% will you reject $H_0$ or not ($z_\alpha = 1.64$ for $\alpha = 0.05$)?

4) A random sample of size $n = 80$ on a population gives the following results

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2.970 | 1.237 | 2.788 | 2.569 | 1.202 | 1.507 | 1.040 | 1.757 | 2.358 | 2.362 |
| 2.505 | 1.012 | 2.249 | 1.253 | 2.235 | 2.542 | 1.373 | 1.993 | 2.019 | 1.632 |
| 2.403 | 1.734 | 2.753 | 1.742 | 2.051 | 1.856 | 2.085 | 2.336 | 1.342 | 2.662 |
| 2.724 | 2.538 | 1.105 | 2.933 | 1.669 | 1.164 | 1.541 | 1.149 | 2.352 | 2.620 |
| 1.928 | 2.138 | 2.024 | 2.173 | 2.478 | 2.743 | 2.156 | 2.929 | 1.667 | 1.786 |
| 1.354 | 1.224 | 2.705 | 2.622 | 2.910 | 2.809 | 1.278 | 2.372 | 1.254 | 1.008 |
| 1.001 | 1.386 | 2.174 | 1.750 | 1.054 | 1.930 | 2.571 | 1.884 | 1.576 | 2.707 |
| 2.052 | 2.457 | 1.533 | 2.567 | 1.936 | 1.394 | 1.894 | 2.457 | 2.932 | 2.514 |

that ordered in ascending order are

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.001 | 1.008 | 1.012 | 1.040 | 1.054 | 1.105 | 1.149 | 1.164 | 1.202 | 1.224 |
| 1.237 | 1.253 | 1.254 | 1.278 | 1.342 | 1.354 | 1.373 | 1.386 | 1.394 | 1.507 |
| 1.533 | 1.541 | 1.576 | 1.632 | 1.667 | 1.669 | 1.734 | 1.742 | 1.750 | 1.757 |
| 1.786 | 1.856 | 1.884 | 1.894 | 1.928 | 1.930 | 1.936 | 1.993 | 2.019 | 2.024 |
| 2.051 | 2.052 | 2.085 | 2.138 | 2.156 | 2.173 | 2.174 | 2.235 | 2.249 | 2.336 |
| 2.352 | 2.358 | 2.362 | 2.372 | 2.403 | 2.457 | 2.457 | 2.478 | 2.505 | 2.514 |
| 2.538 | 2.542 | 2.567 | 2.569 | 2.571 | 2.620 | 2.622 | 2.662 | 2.705 | 2.707 |
| 2.724 | 2.743 | 2.753 | 2.788 | 2.809 | 2.910 | 2.929 | 2.932 | 2.933 | 2.970 |

a) compute the sample median $\tilde{x}$ and fourth spread $f_s$.
b) how may class would you use to draw an histogram of the above result?
c) give the boundaries of each class and the height of the histogram above each class.

Knowing that

$$\sum_{i=1}^{80} x = 160.687 \qquad \sum_{i=1}^{80} x^2 = 350.014$$

f) compute the sample $\bar{x}$ mean and variance $s^2$.
e) compute a 99% CI for the population mean $\mu$ ($z_{\frac{\alpha}{2}} = 2.58$ for $\alpha = 0.01$).