1) The following numbers represent a sample of size $n = 19$ form a given population.

0.985	0.645	0.118	0.894	0.784	0.101	
0.253	−0.321	1.832	0.378	0.679	0.681	
0.753	0.006	0.624	−1.104	0.273	0.126	0.618

a) Compute the sample median and fourth spread.

The ordered results are:

−1.104	−0.321	0.006	0.101	0.118	0.126	
0.253	0.273	0.378	0.618	0.624	0.645	
0.679	0.681	0.753	0.784	0.894	0.985	1.832

so that

$$\tilde{x} = \frac{0.618 + 0.624}{2} = 0.621$$

while

$$lf = \frac{0.118 + 0.126}{2} = 0.122 \qquad uf = \frac{0.681 + 0.753}{2} = 0.717$$

so that

$$f_s = 0.717 - 0.122 = 0.595$$

b) Knowing that $\sum_{i=1} x_i = 8.325$ and $\sum_{i=1} x_i^2 = 10.063$, compute the sample mean and variance.

$$\bar{x} = \frac{1}{n}\sum_{i=1} x_i = \frac{1}{19} \cdot 8.325 = 0.438$$

$$Var(x) = \frac{1}{n-1}\left(\sum_{i=1} x_i^2 - \frac{\left(\sum_{i=1} x_i\right)^2}{n}\right) = \frac{1}{18}\left(10.063 - \frac{8.325^2}{19}\right) = 0.356$$

c) Are there outliers? Extreme outliers?

We have $lf - 1.5f_s = 0.122 - 1.5 \cdot 0.595 = -0.77$, $lf - 3f_s = -1.663$, $uf + 1.5f_s = 1.61$ *and* $uf + 3f_s = 2.502$, *so that* -1.104 *and* 1.832 *are outlier but there are no extreme outlier.*

d) Draw a box plot of the data.

2) In a university every semester 20 students enroll for the honour calculus class for the first time. Each of them as a probability $p = 0.9$ to pass the exam. If they do not pass the exam, they will enroll again in the same class the following semester. In this case it is observed that the probability of passing the exam is $q = 0.5$, i.e. 50% of the students that fail the exam the first time they try will pass it at the second time. If they fails the exam for the second time, they have to drop the class.

a) What is the probability that among the students that enroll for the first time in a given year exactly 4 fail the exam the first time they try it?

The probability that a given student fails is $q = 1 - p = 0.1$ *do that*

$$P(4 student fails) = b(4; 20, 0.1) = \binom{20}{4} 0.1^4 0.9^{16}$$

b) What is the probability that a student will pass the class, i.e. that he will pass the first time or fail the first time and pass the second.

$$\begin{aligned} &P(\text{student pass}) = \\ &= P(\text{student passes 1st test}) + \\ &+ P(\text{student fails 1st test}) P(\text{student passes 2st test} \,|\, \text{student fails 1st test}) = \\ &= 0.9 + 0.1 \cdot 0.5 = 0.95 \end{aligned}$$

c) What is the average number of students that fail the exam twice?

The average number of students that fail the exam twice is 20 minus the average number of students that pass the exam. This last number is $20 \cdot P(student\ pass) = 19$ *so that the average number of students that fail the exam twice is 1.*

Every semester the student attending the class will be formed by the 20 students that just enrolled plus the students that failed the exam the previous semester. Suppose that 3 students failed the exam the previous semester.

d) Choosing at random a student in the class, what is the probability that he already tried the exam once?

There are 23 students in the class and 3 of them already tried the exam once so that the probability is $3/23 = 0.130$

e) You chose one of the 23 student in the class and observe that he passes the exam, what is the probability that he already tried the exam once?

We have that

$$P(\text{tried once} \,|\, \text{passes}) = P(\text{passes} \,|\, \text{tried once}) \frac{P(\text{tried once})}{P(\text{passes})}$$

but

$$\begin{aligned} P(\text{passes}) =& P(\text{passes} \,|\, \text{never tried}) P(\text{never tried}) + P(\text{passes} \,|\, \text{tried once }) P(\text{tried once }) \\ =& 0.9 \cdot 0.87 + 0.5 \cdot 0.13 = 0.848 \end{aligned}$$

so that

$$P(\text{tried once} \,|\, \text{passes}) = 0.5 \cdot \frac{0.13}{0.848} = 0.077$$

3) In a bowl there are 10 red balls, 20 green and 30 blue. You randomly chose 9 out of them without reinsertion.

a) find the probability that 4 of the 9 chosen balls are red.

There are 10 red balls and 50 not-red balls so that:

$$P(\text{4 red balls}) = \frac{\binom{10}{4}\binom{50}{5}}{\binom{60}{9}}$$

b) Find the probability that 3 of the 9 chosen balls are red, 3 are blue and 3 are green.

There are $\binom{10}{3}$ *way to choose 3 red balls,* $\binom{20}{3}$ *way to choose 3 green balls and* $\binom{30}{3}$ *way to choose 3 blue balls so that*

$$P(\text{3 red balls, 3 green balls and 3 blue balls}) = \frac{\binom{10}{3}\binom{20}{3}\binom{30}{3}}{\binom{60}{9}}$$