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Math 4581   First Midterm   Bonetto

A costumer purcahases 10 bulbs from a shop. We know that each bulb has a probability $p = 0.99$ to be working independently from the others.

1) What is the probability that all bulbs are working?
2) What is the probability that exactly 4 out of the 10 bulbs are working?
3) what is the expexted value of the number of working bulbs? and its variance?

A working bulb has a probability of breaking down during the first month of use equal to $q = 0.1$. Let X be the random variable that counts the number of bulbs still working after a month of use among the 10 initially purchased.

4) What is the probability that $X = 8$? what is the p.m.f. of X?

5) Given that after a month one of the bulbs is checked and found not working what is the probability that it was working when purchased?(Hint: let A be the event {*the bulb was working when purchased*} and B the event {*the bulb is working after a month of use*}. Write the conditional probability requested using the product formula.)

Our costumer uses the 10 bulbs in a room where they are connected in series as shown in the figure.
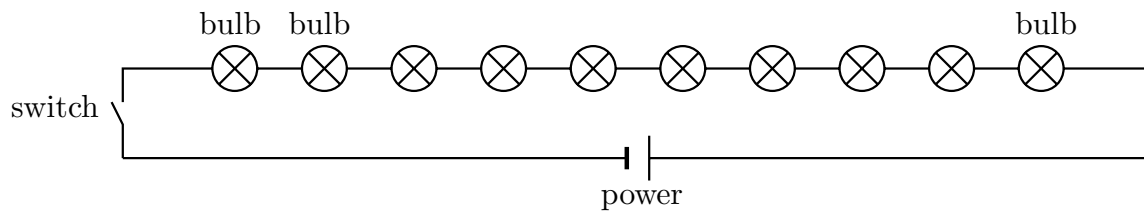
When the switch is closed the bulbs light up only if they are all working, *i.e.* it is enough that one of them is non working and none will light up.

6) what is the probability that the light will not go on when the switch is closed?
7) given that the light does not go on when the switch is closed what is the probability that the first bulb is not working? (Hint: call A_i the event {*the i-th bulb is working*}. Express the events {*the light does not go on*} and {*the first bulb is not working*} in term of the A_i. Write the conditional probability requested explicitely and compute it.)
8) given that the light does not go on when the switch is closed what is the probability that only the first bulb is not working? (Hint: see hint on point 7.)

Assume now that you have two rooms (Room 1 and Room 2) identical to the one described above, *i.e.* each with ten bulbs connected in series. Each room has its own power supply and its own switch. Our costumer buys 20 bulbs and we know that exactly two of them are not working.

9) when both switches are closed, what is the probability that the light will go on only in Room 1? (Hint: let Y be the r.v. counting the number of non working bulbs in Room 1. What kind r.v. is Y? What should be the value of Y be?)

10) when both switches are closed, what is the probability that the light will not go on in all the two rooms, *i.e.* no light in Room 1 and no light in Room 2? (Hint: What should be the value of Y be in this case?)

In the same situation of the previous page assume that 5 out of the twenty bulbs are not working.

11) what is the probability that exactly 3 of the non working bulbs are in Room 1? and the probability that exactly 2 of the non working bulbs are in Room 1? (Hint: see hint to point 9.)

12) when both switches are closed, what is the probability that the light will go on in Room 1?