Spring 07

Math 3770

Name: _____

Test 1

Bonetto

1) The following numbers x_i, $i = 1, \ldots, 18$, represent a sample of size $n = 18$ from a given population.

$$
\begin{array}{cccccc}
2.1389 & 2.8132 & 2.4451 & 2.4660 & 2.6038 & 2.4186 \\
3.8592 & 2.1988 & 2.3529 & 2.2028 & 2.7468 & 1.5104 \\
2.1987 & 2.5252 & 2.8462 & 2.2722 & 2.2026 & 2.0153
\end{array}
$$

a) Compute the sample median and fourth spread and find eventual outliers.

b) Knowing that $\sum_{i=1}^{18} x_i = 43.8166$ and , $\sum_{i=1}^{18} x_i^2 = 110.5081$ compute the sample mean and variance.

d) Draw a box plot of the data.

2) The number of cars that arrive at a control station every day is described by a random variable X with a Poisson p.d.f. with parameter 10, *i.e.* $P(X = x) = \dfrac{10^x}{x!}e^{-10}$. Assume that 40% of all the cars that arrive need service.

   a) Find the expected value and variance of the number of cars that arrive at the control station every day.

   b) Find the probability that exactly N cars arrive and exactly n of these cars need service.

2) Continued

   c) **Bonus** Prove that the number of cars that need service that arrive in a given day is described by a r.v. Y with Poisson distribution with parameter 4.

   c) Using the result of points b) and c) find the probability that exactly N cars arrived in a given day given that exacly n cars needing service arrived that day. Interpret your result in term of the number of car not needing service that arrive in a day.

3) In Atlanta there are 2.000.000 families. Among them 40.000 do not report correctly their incomes. The IRS select a sample of 200 families and controlls their tax returns. Let X be the number of incorect reports among these 200.

    a) What is the probability distribution of X? Write a formula for the probability that $X = 4$.

    b) Use a binomial approximation to compute the average and variance of X. Justify the approximation.

4) Let X be a continuous r.v. with p.d.f. $f(x)$ given by:

$$f(x) = \begin{cases} \dfrac{3}{x^4} & \text{if } x > 1 \\ 0 & \text{otherwise} \end{cases}.$$

Compute:

a) The expected value and variance of X.

b) The c.d.f. of X and the $100p$-percentile.