Practice Test IA for Math 1501, Calculus I

(I): (30 points) Let f be the function defined by

$$f(x) = \frac{x^2+4}{2x} \qquad \text{for} \quad x \neq 0$$

(a) Find all x such that $|f(x) - 2| = 0$.

(b) Find all x such that $|f(x) - 2| < 1/2$.

(c) Find all x such that $|f(x) - 2| < 5$.

(II): (30 points) Which of the following limits exist? If they don't exist, explain why not. If they do, evaluate the limit.

(a) $\lim_{x\to 0} |x| \sin x$

(b) $\lim_{x\to 0} \dfrac{\sin x}{|x|}$

(c) $\lim_{x\to 0} \dfrac{\sin x/x + 2}{\sin x/x + 3}$

(d) $\lim_{x\to 0} \dfrac{1-\cos^2 x}{x^2}$

(III): (20 points)

(a) For which values of A is the following function continuous?

$$f(x) = \begin{cases} (A+1)x^2, & x \leq 1 \\ x/A, & x > 1. \end{cases}$$

(b) Define the following function at $x = 4$ so that it is continuous:

$$f(x) = \frac{\sqrt{x}-2}{\sqrt{x-4}}$$

(IV): (20 points)

(a) Let S be a set of real numbers, and suppose that

$$\text{lub } S = \text{glb } S$$

What does this imply about S? Explain your answer.

(a) Let S and T be non-empty sets of real numbers with $S \subset T$. Which of the following are true, and which are false?

$$\text{lub } S \leq \text{lub } T$$
$$\text{glb } S \leq \text{glb } T$$
$$\text{lub } S \geq \text{glb } T$$
$$\text{glb } S \leq \text{lub } T$$