

Practice Test ID for Math 1501, Calculus I

(I): (30 points) Which of the following limits exist and what is their value. If they don't exist, explain why.

$$(a) \lim_{x \rightarrow 0} \frac{\tan(\sin x)}{x} \quad (b) \lim_{x \rightarrow 0} \frac{|\sin x| + \sin x}{2x}$$
$$(c) \lim_{x \rightarrow 2} \frac{x^{3/2} - 2^{3/2}}{x - 2} \quad (d) \lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 6x + 8}$$

(II): (20 points)

(a) For which numbers $n = 1, 2, \dots$ is $4^{2n+1} + 3^{n+2}$ divisible by 13?

(b) Prove by induction that

$$1 + 2 + 3 + \dots + n = n(n + 1)/2 .$$

(c) Prove by induction that

$$1^3 + 2^3 + 3^3 + \dots + n^3 = n^2(n + 1)^2/4 .$$

(III): (20 points) Let a and b be any positive numbers with $a > b$. Which of the following inequalities are correct? Give counter examples for the incorrect ones.

$$(a) \frac{a}{1+a} > \frac{b}{1+b}$$
$$(b) \frac{b}{1+a} > \frac{a}{1+b}$$
$$(c) ab < \frac{a^2 + b^2}{2}$$

(IV): (30 points) Compute the limit for the following sequences. If the limit does not exist, explain why.

$$(a) a_0 = 1, \quad a_{n+1} = \sin a_n \quad n = 1, 2, \dots .$$

$$(b) \left(1 + \frac{1}{\sqrt{n}}\right)^n, \quad n = 1, 2, \dots .$$

$$(c) \frac{1}{n^2}(1 + 2 + 3 + \dots + n), \quad n = 1, 2, \dots .$$