

Practice Quiz IE for Math 1501, Calculus I

(I): (20 points) Let f be the function defined by

$$f(x) = \frac{2x + 1}{x} \quad \text{for } x \neq 0$$

- (a) Find the largest value of δ so that $|x - 1| < \delta$ implies that $|f(x) - 3| < 1$.
(b) Find the largest value of δ so that $|x - 1| < \delta$ implies that $|f(x) - 3| < 1/2$.

(II): (30 points) Which of the following limits exist? If they don't exist, explain why not. If they do, compute the value of the limit.

(a) $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{\sqrt{x} - 1}$ (b) $\lim_{x \rightarrow \infty} \frac{x^3 + 2}{(x + 1)(2x^2 + 3)}$
(c) $\lim_{x \rightarrow -1} \frac{x^3 + x^2 + x + 1}{x^4 + x^3 + x^2 + x + 1}$ (d) $\lim_{x \rightarrow 0} \frac{(\sin x)^3}{(x + x^2)^3}$

(III): (20 points) Let f be the function defined by

$$f(x) = \frac{2x + 1}{x} \quad \text{for } x \neq 0$$

as in problem I.

- (a) What is its domain?
(b) For each point x that is not in the domain, define $f(x)$ to make it continuous there, or explain why this cannot be done.

(IV): (30 points)

(a) Find a value of N such that

$$\left| \frac{(n + 1) \cos n}{\sqrt{n}(n + 1)} \right| \leq 10^{-4}$$

for all $n \geq N$ but is false for some $n \leq N/2$.

(b) Evaluate

$$\lim_{n \rightarrow \infty} \frac{(n + 1) \cos n}{\sqrt{n}(n + 1)}$$

(c) Evaluate

$$\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n} \right)^{n^2/n+1}$$