Solutions for Practice Test IIA

Problem I:

a)

$$y = \frac{3}{8}x + 1$$

b) The intersection with the x–axis occurs at $x = -8/3$.

Problem II:

a) $f^{'}(x)$ is positive for $0 < x < 2$ and $5 < x < 6$. It is negative for $2 < x < 5$ and zero at $0, 2, 5$.

b) $f^{'}(x)$ is largest at $x = 6$ and smallest at approximately $x = 3.5$.

Problem III:

a) We have to check that

$$\lim_{h \to 0} \frac{f(h) - f(0)}{h} \quad \text{exists} \ .$$

Now

$$\lim_{h \to 0, h > 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0, h > 0} \frac{h^2}{h} = \lim_{h \to 0, h > 0} h = 0 \ .$$

and

$$\lim_{h \to 0, h < 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0, h > 0} \frac{0}{h} = 0 \ .$$

Thus the right limit and the left limit are both zero and hence $f^{'}(0) = 0$.

b)

$$f^{'}(x) = \begin{cases} 2x & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

c) To compute $f^{'}(0)$ we calculate

$$\lim_{h \to 0, h > 0} \frac{f^{'}(h) - f^{'}(0)}{h} = \lim_{h \to 0, h > 0} \frac{2h}{h} = \lim_{h \to 0, h > 0} 2 = 2 \ .$$

and

$$\lim_{h \to 0, h < 0} \frac{f^{'}(h) - f^{'}(0)}{h} = \lim_{h \to 0, h > 0} \frac{0}{h} = 0 \ .$$

Since these two limits are different the derivative does not exist.

Problem IV: If D denotes the distance we get

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{6}{\sqrt{2}} \quad \text{units per second} \ .$$

Problem V

a)

$$2$$

.

b)

$$2x^3(1+\tan(x^2))+2x\tan(x^2)\ .$$

c)

$$\frac{1}{2\sqrt{x+\sqrt{x}}}(1+\frac{1}{2\sqrt{x}})\ .$$

d)

$$2xe^{x^2}\ .$$