

Practice Quiz IIIA for Math 1501, Calculus I

(I): (30 points) A disk of paper is to be formed into a conical cup as follows: A wedge is cut out and removed along two radii. The two edges thus formed are then joined without overlap.

Let x be the fraction of the disk that is removed. What should x be for the volume of the cup to be maximized, and what is the maximal volume?

(Recall that the volume of a right cylindrical cone with radius r and height h is $\frac{\pi}{3}r^2h$.)

(II): (30 points) Let f be the function

$$f(x) = \frac{x^2\sqrt{x^2+1}}{1-x^2}.$$

- (a) Find all asymptotes of this function, if any.
- (b) Find all critical numbers for this function, if any.
- (c) Find all local minima for this function, if any.
- (d) Find all local maxima for this function, if any.
- (e) Does the function have an absolute minimum?
- (f) Does the function have an absolute maximum?
- (g) Sketch the graph of this function.

(III): (25 points) Let f be a function that satisfies:

$$f(1) = 1 \quad f(4) = 2 \quad \text{and} \quad |f'(x)| \leq 2 \quad \text{for all} \quad 1 \leq x \leq 2.$$

- (a) Could it be true that $f(2) = 0$?
- (b) Could it be true that $f(2) = -1$?
- (c) Could it be true that $f(2) = -2$?
- (d) Could it be true that $f(2) = 5$?
- (e) Could it be true that $f(3) = 5$?
- (f) Could it be true that $f(3) = 0$?

(IV): (15 points) The function

$$g(x) = \frac{A}{2x^2} + \frac{Bx^3}{3} \quad \text{for} \quad x > 0$$

has a minimum at $x = 1$, and the minimum value is 1. What are the values of A and B ?