

Practice Quiz IIIB for Math 1501, Calculus I

(I): (30 points)

(a) Among all pairs of positive numbers a and b satisfying

$$a + b = 4 ,$$

which pair maximizes the quantity ab^2 ?

(b) Among all pairs of positive numbers a and b satisfying

$$a + b = 4 ,$$

which pair maximizes the quantity $(\frac{1}{a} + \frac{1}{b})^{-1}$?

(II): (30 points) Let f be the function

$$f(x) = \sqrt{x^2 + 2x} - \sqrt{x^2 + 2} \quad \text{for } x > 0 .$$

- (a) Find all asymptotes of this function, if any.
- (b) Find all critical numbers for this function, if any.
- (c) For which values of x , if any, is this function increasing?
- (d) For which values of x , if any, is this function decreasing?
- (e) Does the function have an absolute maximum?
- (f) Sketch the graph of this function.

(III): (25 points) Let f be a function that satisfies:

$$f(1) = 1 \quad f'(1) = -1 \quad \text{and} \quad |f''(x)| \leq 1 \quad \text{for all } 1 \leq x \leq 2 .$$

- (a) Could it be true that $f(2) = 2$?
- (b) Could it be true that $f(2) = 1$?
- (c) Could it be true that $f(2) = -2$?
- (d) Could it be true that $f(4) = -2$?
- (e) Could it be true that $f(4) = 1$?

(IV): (15 points) Let f be a function satisfying

$$f(2) < 2 \quad f(3) > 3$$

and

$$|f'(x)| \leq \frac{1}{2} \quad \text{for all } 2 \leq x \leq 3 .$$

Define

$$a_0 = 2 \quad \text{and} \quad a_{n+1} = f(a_n) \quad \text{for all } n \geq 0 .$$

What is the smallest value of n such that we can be sure that

$$|a_n - L| \leq 10^{-3}$$

where L is the solution of $x = f(x)$ in the interval $(2, 3)$?