

TEST 4

Math 2403 C2

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Name _____

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Show your work and justify your answers to receive credit! Work neatly. Put a box around your answer.

1. (20 pts.) The point $(0, 2)$ is a critical point of the system

$$\begin{aligned}x' &= 4x - 4x^2 - xy \\y' &= 6y - 3y^2 - 5xy\end{aligned}\quad (*)$$

- a. Find the linearization of the system $(*)$ at $(0, 2)$.
- b. Describe the type and stability of the critical point $(0, 2)$ of $(*)$. Justify.
2. (20 points) Find the Laplace transform of the solution of $x''' + 2x'' - 4x = t^5$, $x(0) = 2$, $x'(0) = 0$, $x''(0) = -3$. (You don't need to find the actual solution of the differential equation!)

3. (20 points) Solve the initial value problem $x'' - 16x = \delta(t - 3)$, $x(0) = 7$, $x'(0) = 0$.

4. (10 pts.) Find the Laplace transform of $f(t)$ for $f(t) = u_3(t)\sin(2\pi t)$

5. (15 points) Find the Laplace transform of a nontrivial solution of $tx'' - 2x' = 0$, $x(0) = 0$. (You don't need to find the actual solution of the differential equation!)

6. (15 points) Consider the equation $x'' - 9x = g(t)$, $x(0) = x'(0) = 0$, where $g(t)$ is piecewise continuous and $|g(t)|$ is bounded by Me^{ct} as $t \rightarrow \infty$, $c > 0, M > 0$. Apply the convolution theorem to write the solution as an integral. (You do not need to evaluate the integral.)