

TEST 1

Name _____

Math 2403 C3

February 3, 2000

Show your work and justify your answers to receive credit!

1. (20 pts.) Solve $\left(y^3 - \frac{6}{x^2} + 2xe^y\right) dx + (3y^2x + 5 \cos y + x^2e^y) dy = 0$, $y(2) = 0$.

2. (20 pts.) Find the general solution of the differential equation $xy' - 3y = -6x^2$, $x > 0$.

3. (10 pts.) Without solving the differential equation, determine the largest interval on which the initial value problem is guaranteed to have a unique solution. Briefly explain your answer. (Do not attempt to find the solution!)

$$(x - 3)y' + \frac{x - 7}{x + 5}y = \frac{4}{x - 9}, \quad y(4) = 1$$

4. (20 pts.) Consider the differential equation $\frac{dP}{dt} = 2P(P - 3)(P - 5)^2$, $t \geq 0$.

(One does not need to solve this differential equation to complete the problem.)

- Find the equilibrium solutions. Classify each as stable, unstable, or semi-stable.
- Sketch label solution curves corresponding to the solutions with initial condition $P(0) = 2$ and $P(0) = 4$.

5. (10 pts.) Consider the initial value problem $y' = x - 2y$, $y(1) = 3$. Use Euler method with $h = 0.1$ to determine approximate values of the solution $y(x)$ at $x = 1.1$ and $x = 1.2$.

6. (20 pts.) A vat contains 400 gallons of solution consisting of water and 50 lbs. of salt. Assume that water containing $1/2$ lbs. of salt per gallon is entering the vat at a rate of 5 gallons per minute, and that the well-stirred mixture is leaving the vat at the rate of 6 gallons per minute.

- a. Set up the appropriate differential equation with initial condition to model this situation.
- b. Find the concentration of the salt in the solution in the vat after 20 minutes.