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A Database Schema for the Analysis of Global Dynamics of Multiparameter Systems*

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Abstract. A generally applicable, automatic method for the efficient computation of a database of global dynamics of a multiparameter dynamical system is introduced. An outer approximation of the dynamics for each subset of the parameter range is computed using rigorous numerical methods and is represented by means of a directed graph. The dynamics is then decomposed into the recurrent and gradient-like parts by fast combinatorial algorithms and is classified via Morse decompositions. These Morse decompositions are compared at adjacent parameter sets via continuation to detect possible changes in the dynamics. The Conley index is used to study the structure of isolated invariant sets associated with the computed Morse decompositions and to detect the existence of certain types of dynamics. The power of the developed method is illustrated with an application to the two-dimensional density-dependent Leslie population model. An interactive visualization of the results of computations discussed in the paper can be accessed at the Web site http://chomp.rutgers.edu/database/, and the source code of the software used to obtain these results has also been made freely available.

Key words. database, dynamical system, Conley index, Morse decomposition, Leslie population models, combinatorial dynamics, multiparameter system

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