Learning Goals

Learning goals articulate what students are **expected to be able to do** in a course that can be measured. This course has **course-level** learning goals that pertain to the entire course, and **section-level** learning goals that pertain to sections of the course.

Course-Level Learning Goals

By the end of this course, it is expected that students will be able to do the following.

A) Solve systems of linear questions.

B) Solve eigenvalue problems.

C) Analyze mathematical statements and expressions (for example, to assess whether a particular statement is accurate, or to describe solutions of systems in terms of existence and uniqueness).

D) Write logical progressions of precise mathematical statements to justify and communicate your reasoning.

E) Apply linear algebra concepts to model, solve, and analyze real-world situations.

Section-Level Learning Goals

Section-level learning goals pertain to specific sections of our textbook. They state what students are expected to be able to do specifically for the topics in each section.

1 Linear Equations

1.1 Systems of Linear Equations; Lines and Planes in $\mathbb{R}^n$

1. Visualize and parametrize lines and planes in $\mathbb{R}^3$, and extend ideas to $\mathbb{R}^n$.
2. Apply elementary row operations to solve linear systems of equations.
3. Make the connection between solutions of linear systems and intersections of lines and planes.
4. Characterize a linear system in terms of the number of solutions, and whether the system is consistent or inconsistent.
5. Express a set of linear equations as an augmented matrix.

1.2 Row Reduction and Echelon Forms

1. Characterize a linear system in terms of the number of leading entries, free variables, pivots, pivot columns, pivot positions.
2. Apply the row reduction algorithm to reduce a linear system to echelon form, or reduced echelon form.
3. Apply the row reduction algorithm to compute the coefficients of a polynomial.

1.3 Vector Equations

1. Apply geometric and algebraic properties of vectors in $\mathbb{R}^n$ to compute vector additions and scalar multiplications.
2. Characterize a set of vectors in terms of **linear combinations**, their **span**, and how they are related to each other geometrically.
3. Characterize the solutions to a linear system in terms of linear combinations and span.

1.4 The Matrix Equation

1. Compute matrix-vector products.
2. Express linear systems as vector equations and matrix equations.
3. Characterize linear systems and sets of vectors using the concepts of span, linear combinations, and pivots.

1.5 Solution Sets of Linear Systems

1. Express the solution set of a linear system in parametric vector form.
2. Provide a geometric interpretation to the solution set of a linear system.
3. Characterize homogeneous linear systems using the concepts of free variables, span, pivots, linear combinations, and echelon forms.

1.6 Applications of Linear Systems

1. Set up, and solve, matrix equations to find equilibrium in a Leontief input-output model from economics.
2. Balance chemical equations using linear systems.
3. Use linear equations to understand and solve problems in network flow.

1.7 Linear Independence

1. Characterize a set of vectors and linear systems using the concept of linear independence.
2. Construct dependence relations between linearly dependent vectors.

1.8 An Introduction to Linear Transforms

1. Construct and interpret linear transformations in $\mathbb{R}^2$ or $\mathbb{R}^3$ (for example, interpret a linear transform as a projection, or as a shear).
2. Characterize linear transforms using the concepts of existence and uniqueness.

1.9 Linear Transforms

1. Identify and construct linear transformations of a matrix.
2. Characterize linear transformations as onto, one-to-one.
3. Solve linear systems represented as linear transforms.
4. Express linear transforms in other forms, such as as matrix equations, and vector equations.

2 Matrix Algebra

2.1 Matrix Operations

1. Apply matrix algebra, the matrix transpose, and the zero and identity matrices, to solve and analyze matrix equations

2.2 Inverse of a Matrix

1. Apply the formal definition of an inverse, and its algebraic properties, to solve and analyze linear systems.
2. Compute the inverse of an $n \times n$ matrix, and use it to solve linear systems.

2.3 Invertible Matrices

1. Characterize the invertibility of a matrix using the Invertible Matrix Theorem.

2.8 Subspaces of $\mathbb{R}^n$

1. Determine whether a set is a subspace.
2. Determine whether a vector is in a particular subspace, or find a vector in that subspace.
3. Construct a basis for a subspace (for example, a basis for Col(A)).

2.9 Dimension and Rank

1. Calculate the coordinates of a vector in a given basis.
2. Characterize a subspace using the concept of dimension.
3. Characterize a matrix using the concepts of rank, column space, null space.
4. Apply the Rank, Basis, and Matrix Invertibility theorems to describe matrices and subspaces.

3 Determinants

3.1 Introduction to Determinants

1. Compute determinants of $n \times n$ matrices using a cofactor expansion.
2. Apply theorems to compute determinants of matrices that have particular structures.

3.2 Properties of the Determinant

1. Apply properties of determinants (related to row reductions, transpose, and matrix products) to compute determinants.
2. Use determinants to determine whether a square matrix is invertible.

5 Eigenvalues and Eigenvectors

5.1 Eigenvectors and Eigenvalues

1. Verify that a given vector is an eigenvector of a matrix.
2. Verify that a scalar is an eigenvalue of a matrix.
3. Construct an eigenspace for a matrix.
4. Apply theorems related to eigenvalues (for example, to characterize the invertibility of a matrix).

5.2 The Characteristic Equation

1. Construct the characteristic polynomial of a matrix and use it to identify eigenvalues and their multiplicities.
2. Characterize the long-term behaviour of dynamical systems using eigenvalue decompositions.

5.3 Diagonalization

1. Determine whether a square matrix can be diagonalized.
2. Diagonalize square matrices.
3. Apply diagonalization to compute matrix powers.

5.5 Complex Eigenvalues

1. Appendix B: Add, subtract, multiply, and divide complex numbers.
2. Appendix B: Find the conjugate and modulus of a complex number.
3. Appendix B: Polar coordinates, Fundamental Theorem of Algebra.
4. Diagonalize 2×2 matrices that have complex eigenvalues.
5. Use eigenvalues to determine identify the rotation and dilation of a linear transform.
6. Apply theorems to characterize matrices with complex eigenvalues.

5.6 Discrete Dynamical Systems

1. Plot trajectories of discrete dynamical systems.
2. Classify the origin as an attractor, repeller, or saddle point.
3. Find directions of greatest attraction and repulsion.
4. Solve real-world examples using dynamical systems.

6 Orthogonality and Least Squares

6.5 Least-Squares Problems

1. Understand the connection between least squares solutions and projections onto a particular subspace.
2. Compute general solutions, and least squares errors, to least squares problems using the normal equations.

10 Finite-State Markov Chains (combination of 10.1 and 10.2)

1. Find steady state vectors for stochastic matrices and interpret them in real-world terms.
2. Understand Google page rank as an application of stochastic matrices.