Math 1553 Supplement §2.8, 2.9

1. Find bases for the column space and the null space of

$$A = \begin{pmatrix} 0 & 1 & -3 & 1 & 0 \\ 1 & -1 & 8 & -7 & 1 \\ -1 & -2 & 1 & 4 & -1 \end{pmatrix}.$$

2. Which of the following are subspaces of $\mathbf{R}^4$? Why or why not?

(a) $V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \,\middle|\, x + y = 0 \text{ and } z + w = 0 \right\}$ (b) $W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \,\middle|\, xy - zw = 0 \right\}$

3. For (a), answer "YES" if the statement is always true, "NO" if it is always false, and "MAYBE" otherwise.

a) If A is an $m \times n$ matrix and $\mathrm{Nul}(A) = \mathbf{R}^n$, then $\mathrm{Col}(A) = \{0\}$.

YES NO MAYBE

b) Give an example of 2×2 matrix whose column space is the same as its null space.

4. Let $\mathcal{B} = \left\{ \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right\}$, and suppose $[x]_{\mathcal{B}} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$. Find x, and draw a picture which clearly represents x as a linear combination of $b_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $b_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

5. Go back to the 2.8-2.9 worksheet, #3: Find a vector b_3 such that $\{b_1, b_2, b_3\}$ is a basis of $\mathbf{R}^3$.