Math 1553 Supplement §5.3 (with some more practice from §5.2)

For those who want additional practice problems after completing the worksheet, here are some extra practice problems.

1. Let A and B be 3×3 real matrices. Answer yes / no / maybe:

   a) If A and B have the same eigenvalues, then A is similar to B.

   b) If A is diagonalizable and invertible, then A^{-1} is diagonalizable.

   c) If A and B are invertible and A is similar to B, then A^{-1} is similar to B^{-1}.

2. Let $A = \begin{pmatrix} 8 & 36 & 62 \\ -6 & -34 & -62 \\ 3 & 18 & 33 \end{pmatrix}$.

The characteristic polynomial for A is $-\lambda^3 + 7\lambda^2 - 16\lambda + 12$, and $\lambda - 3$ is a factor. Decide if A is diagonalizable. If it is, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

3. Give an example of a non-diagonal 2×2 matrix which is diagonalizable but not invertible. Justify your answer.