MATH 1553, SPRING 2018
SAMPLE MIDTERM 2 (VERSION B), 1.7 THROUGH 2.9

Name		**Section**	

1	2	3	4	5	Total

Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (calculator, notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. I recommend completing the practice exam in 50 minutes, without notes or distractions.

The exam is not designed to test material from the previous midterm on its own. However, knowledge of the material prior to section §1.7 is necessary for everything we do for the rest of the semester, so it is fair game for the exam as it applies to §§1.7 through 2.9.

Problem 1.

[2 points each]

In what follows, A is a matrix, and $T(x) = Ax$ is its matrix transformation.

Circle **T** if the statement is always true, and circle **F** otherwise. You do not need to explain your answer.

a) **T** **F** The zero vector is in the range of T.

b) **T** **F** If A is a non-invertible square matrix, then two of the columns of A are scalar multiples of each other.

c) **T** **F** If A is a 2×5 matrix, then Nul$A$ is a subspace of $\mathbf{R}^2$.

d) **T** **F** If A has more columns than rows, then T is not onto.

e) **T** **F** If T is one-to-one and onto, then A is invertible

Problem 2.

[2 points each]

Which of the following are subspaces of $\mathbf{R}^4$ and **why?**

a) Span $\left\{ \begin{pmatrix} 1 \\ 0 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ 7 \\ 9 \\ 13 \end{pmatrix}, \begin{pmatrix} 144 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

b) Nul $\begin{pmatrix} 2 & -1 & 3 \\ 0 & 0 & 4 \\ 6 & -4 & 2 \\ -9 & 3 & 4 \end{pmatrix}$

c) Col $\begin{pmatrix} 2 & -1 & 3 \\ 0 & 0 & 4 \\ 6 & -4 & 2 \\ -9 & 3 & 4 \end{pmatrix}$

d) $V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; xy = 2zw \right\}$

e) The range of a linear transformation with codomain $\mathbf{R}^4$.

Problem 3.

Consider the matrix A and its reduced row echelon form:

$$\begin{pmatrix} 2 & 4 & 7 & -16 \\ 3 & 6 & -1 & -1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{pmatrix}.$$

a) [4 points] Find a basis $\{v_1, v_2\}$ for $\operatorname{Col} A$.

b) [3 points] What are $\operatorname{rank} A$ and $\dim \operatorname{Nul} A$?

c) [3 points] Find a basis $\{w_1, w_2\}$ for $\operatorname{Col} A$, such that w_1 is a not scalar multiple of v_1 or v_2, and likewise for w_2. Justify your answer.

Problem 4.

[5 points each]

Consider the vectors

$$v_1 = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \qquad v_2 = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} \qquad v_3 = \begin{pmatrix} 1 \\ h \\ 5 \end{pmatrix}.$$

a) Find the value of h for which $\{v_1, v_2, v_3\}$ is linearly *de*pendent.

b) For this value of h, produce a linear dependence relation among v_1, v_2, v_3.

Problem 5.

Consider the matrices

$$A = \begin{pmatrix} 1 & -2 \\ 0 & 2 \\ 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 3 & 2 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{pmatrix}.$$

Let T and U be the associated linear transformations, respectively

$$T(x) = Ax \qquad U(x) = Bx.$$

a) [2 points] Fill in the boxes:

$$T : \mathbf{R}^{\square} \longrightarrow \mathbf{R}^{\square} \qquad U : \mathbf{R}^{\square} \longrightarrow \mathbf{R}^{\square}.$$

b) [2 points] Is T one-to-one?

c) [3 points] Find the standard matrix for U^{-1}.

d) [3 points] Find the standard matrix for $U \circ T$.

[Scratch work]