Name:______________________________ Recitation Section:__________

Math 1553 Lecture C, Quiz 5: 2.1, 2.2, 2.3 (10 points, 10 minutes)

Solutions

Show your work and justify answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit.

1. (1 point each) True or False. Circle TRUE if the statement is *always* true. Otherwise, circle FALSE. You do not need to justify your answer on this problem.

a) If A is a 3×3 matrix and $A\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, then there is a vector in $\mathbf{R}^3$ that cannot be written as a linear combination of the columns of A.

[TRUE] FALSE

(IMT: non-trivial solution to $Ax = 0 \implies$ columns of A don't span R^3.)

b) If A is an invertible $n \times n$ matrix, then $(A^{-1})^{-1} = A$. **[TRUE]** FALSE

c) If $T : \mathbf{R}^n \to \mathbf{R}^n$ is a linear transformation that is not invertible, then there are vectors x and y in $\mathbf{R}^n$ so that $x \neq y$ but $T(x) = T(y)$.

[TRUE] FALSE (IMT: T is not invertible $\implies$ T is not one-to-one)

d) If A is a 3×4 matrix and B is a 4×2 matrix, then the linear transformation transformation Z defined by $Z(x) = ABx$ has domain $\mathbf{R}^2$ and codomain $\mathbf{R}^3$.

[TRUE] FALSE (AB is a 3×2 matrix)

2. (3 points) Let $A = \begin{pmatrix} 4 & -1 \\ 7 & 1 \end{pmatrix}$. Find A^{-1}.

Solution.

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $A^{-1} = \dfrac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, so

$$A^{-1} = \frac{1}{4 - (-7)}\begin{pmatrix} 1 & 1 \\ -7 & 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{11} & \frac{1}{11} \\ \frac{-7}{11} & \frac{4}{11} \end{pmatrix}.$$

(Turn over to the back side for problem 3!)

3. (3 points) Write two square matrices A and B so that $AB \neq BA$. Demonstrate that $AB \neq BA$ by computing AB and BA.

Solution.

Many examples. We did one in the handwritten notes in class and there is a different example on the PDF slides. Here is a third:

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}.$$

$$AB = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \qquad BA = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$