Midterm 1 (1.1-2.4), Solutions

1. Answer true or false.

   (a) If the augmented matrix corresponding to a system of linear equations has a pivot in every row, then the system must be consistent.

   (b) If a linear system of 3 equations in 3 variables is consistent, then it must have exactly one solution.

   **Solution**: (a) False. This was taken from a Webwork. For example, $\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right)$.

   (b) False. For example, consider the system corresponding to $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$. This has infinitely many solutions.

2. The Island Juice Bar makes three kinds of juices. A serving of their tropical juice uses 1 cup of pineapple juice and 4 cups of orange juice, a serving of their summer juice uses 3 cups of pineapple juice and 1 cup of orange juice, and a serving of their beach juice uses 1 cup of pineapple juice and 2 cups of orange juice. Which of the following systems of equations describes the number of servings of tropical juice (T), summer juice (S), and beach juice (B) that the Island Juice Bar can make if they have 57 cups of pineapple juice and 55 cups of orange juice?

   (a)
   $$\begin{aligned} T + 3S + B &= 57 \\ 4T + S + 2B &= 55 \end{aligned}$$

   (b)
   $$\begin{aligned} T + 4T &= 112 \\ 3S + S &= 112 \\ 2B + 2B &= 112 \end{aligned}$$

   (c)
   $$\begin{aligned} T + 3T &= 55 \\ 3S + S &= 57 \\ 2B + 2B &= 112 \end{aligned}$$

   (d)
   $$\begin{aligned} T + 4S + 2B &= 57 \\ 3T + S + B &= 55 \end{aligned}$$

   (e)
   $$\begin{aligned} T + 3S + B &= 112 \\ 4T + S + 2B &= 112 \end{aligned}$$

   **Solution**: For pineapple juice, we have 57 cups to use:

   $$T(1) + S(3) + B(1) = 57.$$

   For orange juice, we have 55 cups to use:

   $$T(4) + S(1) + B(2) = 55.$$

   The system of equations is therefore option (a):

   $$T + 3S + B = 57$$

   $$4T + S + 2B = 55.$$

3. Which of the following are in reduced row echelon form (RREF)? Select all that apply.

   (a) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

   (b) $\left(\begin{array}{cc|c} 1 & 5 & 1 \\ 0 & 0 & 1 \end{array}\right)$

   (c) $\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \end{array}\right)$

   (d) $\left(\begin{array}{cc|c} 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right)$

   **Solution**: (a) is not. We would need to swap its rows to put it in RREF.

   (b) is not. Its pivot in the 2nd row has a "1" entry above it.

   (c) is in RREF.

   (d) is in RREF.

4. How many pivots does the matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 3 & 1 \end{pmatrix}$ have?

   **Solution**: This question is a bit ambiguous, so every student in Math 1553 was given credit.

   One definition is that a pivot is the first nonzero entry in a row of any matrix, while the appropriate definition is that a pivot is the first nonzero entry in a row *of a matrix in row echelon form.* This problem was written with the intention of using the second definition, which would give 2 pivots: Subtracting $R_1$ from $R_2$ gives us

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 3 & 1 \end{pmatrix},$$

   and swapping the last two rows gives us

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

5. Find all values of $a$ so that the following matrix is in reduced row echelon form. $\begin{pmatrix} 1 & -2 & -1 \\ 0 & a & 1 \\ 0 & 0 & 0 \end{pmatrix}$

   (a) $a = 0$ only
   (b) $a = 1$ only
   (c) $a = 0$ and $a = 1$
   (d) Every real number $a$
   (e) There is no value of $a$ so that the matrix is in reduced row echelon form

   **Solution**: If the matrix has any hope of being in RREF, then $a$ must be 0 (otherwise, $a$ would correspond to a pivot with a nonzero entry above it).

   However, if $a = 0$ then the matrix is not in RREF because the matrix would be

$$\begin{pmatrix} 1 & -2 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$

   which has a nonzero entry above its rightmost pivot.

   Therefore, there is no value of $a$ for which the matrix is in RREF.

6. In each case, answer whether the linear system of equations corresponding to the augmented matrix is consistent or inconsistent.

   (a) $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 5 \\ 0 & 0 & 1 & 0 \end{array}\right)$

   (b) $\left(\begin{array}{ccc|c} 0 & 1 & 3 & 2 \\ 1 & 2 & 3 & 5 \end{array}\right)$

   **Solution**: (a) Subtracting the first row from the second gives second row $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 1 \end{array}\right)$, so the system is inconsistent.

   (b) The system is consistent. Switching the first two rows, we can see that the matrix has two pivots and they are both to the left of the vertical bar.

7. Suppose that we are given a consistent system of four linear equations in three variables, and suppose that the augmented matrix corresponding to the system has two pivots. Which one of the following describes the solution set for the system, geometrically?

   (a) a point in $\mathbf{R}^4$
   (b) a line in $\mathbf{R}^4$
   (c) a plane in $\mathbf{R}^4$
   (d) a point in $\mathbf{R}^3$
   (e) a line in $\mathbf{R}^3$
   (f) a plane in $\mathbf{R}^3$
   (g) We need more information to determine the geometry of the solution set

   **Solution**: All of Math 1553 was given credit due to the "pivot" ambiguity mentioned in problem 4. We will write the solution using the definition of pivot as first nonzero entry in a row echelon form of a matrix. Our solution set involves three variables, so it will live in $\mathbf{R}^3$. Since the corresponding augmented matrix for this consistent system has two pivots, that means that exactly one of the three columns to the left of the vertical bar will not have a pivot, se we get exactly 1 free variable.

   Therefore, the solution set is a line in $\mathbf{R}^3$.

8. Find all values of $h$ and $k$ so that the system below has infinitely many solutions.

$$x + hy = 4$$
$$4x - 12y = k.$$

(a) $h = 3$ and $k = 16$
(b) $h \neq 3$ and $k = 16$
(c) $h = -3$ and $k = 16$
(d) $h \neq -3$ and $k = 16$
(e) $h = -3$ and $k$ is any real number
(f) $h = 3$ and $k$ is any real number
(g) $h \neq -3$ and $k$ is any real number
(h) $h \neq 3$ and $k$ is any real number

**Solution**:

$$\left(\begin{array}{cc|c} 1 & h & 4 \\ 4 & -12 & k \end{array}\right) \to \left(\begin{array}{cc|c} 1 & h & 4 \\ 0 & -12-4h & k-16 \end{array}\right).$$

For this to have infinitely many solutions, we must have $-12 - 4h = 0$ (otherwise we'd have a unique solution) and then we conclude $k - 16 = 0$ (otherwise we would have a pivot in the rightmost column). This gives $h = -3$ and $k = 16$.

We could have also argued geometrically by noting that these two lines defined by the equations will have infinitely many points in common precisely when they are the same line, which means $h = -3$ and $k = 16$.

9. Suppose we are given a system of two linear equations in three variables. Which of the following are possibilities for the set of solutions? Select all that apply.

(a) No solution
(b) a point in $\mathbf{R}^3$
(c) a line in $\mathbf{R}^3$
(d) a plane in $\mathbf{R}^3$

**Solution**: (a), (c), and (d) are possible.

For (a), consider the system corresponding to $\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$.

Note (b) is impossible: in order for the solution set to consist of a single point, we would need 3 pivots to the left of the vertical bar in the corresponding augmented matrix (in order to have no free variables), which is impossible since it would only have 2 rows.

For (c), consider $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right)$.

For (d), consider $\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right)$.

10. This problem has two unrelated parts.

   (a) Consider the system of equations below:

$$x + y = 1$$

$$x - y = 0.$$

   True or false: The solution set to this system of equations given below has parametric form

$$x = 1 - y, \quad y = y \ \text{(any real number)}.$$

   (b) Is it possible for the span of 3 vectors in $\mathbf{R}^4$ to be a line?

   **Solution**: (a) False. The system does not have infinitely many solutions. Instead, it has the unique solution $x = 1/2$ and $y = 1/2$.

   (b) Yes. For example, $\text{Span}\left\{\begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}2\\0\\0\\0\end{pmatrix}, \begin{pmatrix}3\\0\\0\\0\end{pmatrix}\right\}$. This is the same as $\text{Span}\left\{\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\right\}$, which is the line through the origin and $\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ in $\mathbf{R}^4$.

11. Solve the vector equation: $x\begin{pmatrix}1\\2\end{pmatrix} + y\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}5\\12\end{pmatrix}$.

   **Solution**: $\left(\begin{array}{cc|c}1 & 1 & 5\\2 & 3 & 12\end{array}\right) \to \left(\begin{array}{cc|c}1 & 1 & 5\\0 & 1 & 2\end{array}\right) \to \left(\begin{array}{cc|c}1 & 0 & 3\\0 & 1 & 2\end{array}\right)$, so $x = 3$ and $y = 2$.

12. Let $v_1 = \begin{pmatrix}1\\0\\-1\end{pmatrix}$, $v_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}$ and $v_3 = \begin{pmatrix}-3\\-3\\3\end{pmatrix}$. Which one of the following vectors is *not* in $\text{Span}\{v_1, v_2, v_3\}$?

   (a) $\begin{pmatrix}3\\2\\-3\end{pmatrix}$

   (b) $\begin{pmatrix}0\\-\sqrt{2}\\0\end{pmatrix}$

   (c) $\begin{pmatrix}1\\0\\0\end{pmatrix}$

   (d) $\begin{pmatrix}0\\0\\0\end{pmatrix}$

   **Solution**: We don't need to do any work to conclude that the vectors in (b) and (d) are in $\text{Span}\{v_1, v_2, v_3\}$, since $\begin{pmatrix}0\\-\sqrt{2}\\0\end{pmatrix}$ is a scalar multiple of $v_2$ and the zero vector is automatically in $\text{Span}\{v_1, v_2, v_3\}$.

With this in mind, the answer must be (a) or (c). For the vector in (c), row-reduction quickly gives

$$\left(\begin{array}{ccc|c} 1 & 0 & -3 & 1 \\ 0 & 1 & -3 & 0 \\ -1 & 0 & 3 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -3 & 1 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

which is inconsistent, so $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ is not in $\mathrm{Span}\{v_1, v_2, v_3\}$.

13. Find all values of $h$ so that $\begin{pmatrix} 3 \\ -1 \\ h \end{pmatrix}$ is a linear combination of $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

   (a) All real $h$ except 0
   (b) All real $h$ except 1
   (c) All real $h$ except $-1$
   (d) All real $h$ except 3
   (e) $h = 0$ only
   (f) $h = 1$ only
   (g) $h = -1$ only
   (h) $h = 3$ only

**Solution**: We row-reduce:

$$\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & -1 \\ 1 & 3 & h \end{array}\right) \xrightarrow{R_3 = R_3 - R_1} \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & -1 \\ 0 & 2 & h-3 \end{array}\right) \xrightarrow{R_3 = R_3 - 2R_2} \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & h-1 \end{array}\right).$$

This is consistent if and only if there is not a pivot in the rightmost column, so $h - 1 = 0$, thus $h = 1$.

14. True or false.

   (a) For every real number $d$, the span of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} d \\ -3 \end{pmatrix}$ is $\mathbf{R}^2$.

   (b) Suppose $v_1$, $v_2$, and $v_3$ are vectors in $\mathbf{R}^3$ with the property that $\mathrm{Span}\{v_1, v_2\}$ is a plane and that $v_3$ is not a linear combination of $v_1$ and $v_2$. Then $\mathrm{Span}\{v_1, v_2, v_3\}$ must be all of $\mathbf{R}^3$.

**Solution**: (a) True. The matrix $\begin{pmatrix} 1 & d \\ 0 & -3 \end{pmatrix}$ automatically has a pivot in each row no matter what $d$ is, so its columns span $\mathbf{R}^2$.

(b) True. $\mathrm{Span}\{v_1, v_2\}$ is a plane, so $\begin{pmatrix} v_1 & v_2 \end{pmatrix}$ has two pivots. Since $v_3$ is not in $\mathrm{Span}\{v_1, v_2\}$, the augmented system $\left(\begin{array}{cc|c} v_1 & v_2 & v_3 \end{array}\right)$ is inconsistent so it has a pivot in the rightmost column, for a total of 3 pivots (a pivot in each row since the vectors are in $\mathbf{R}^3$). This means $\begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$ has a pivot in each row, so its columns span $\mathbf{R}^3$.

15. Select the correct answer in each case.

   (a) Select the correct answer to fill in the blank: In a matrix equation $Ax = b$, the number of ________ in $A$ must be equal to the number of entries in $b$.

   (b) Suppose $A$ is a $2 \times 3$ matrix. Is it possible for the matrix equation $Ax = b$ to be consistent for every $b$ in $\mathbf{R}^2$?

**Solution**: (a) For a matrix $A$ with $m$ rows and $n$ columns, the equation $Ax = b$ requires $x$ in $\mathbf{R}^n$ and $b$ in $\mathbf{R}^m$. In other words, the number of rows of $A$ must match the number of entries in $b$.

(b) Yes, it is possible. For example, if

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

then $A$ has a pivot in each row so $Ax = b$ is consistent for each $b$ in $\mathbf{R}^2$.

16. Suppose $A$ is a $11 \times 7$ matrix and $b$ is a vector so that the matrix equation $Ax = b$ has exactly one solution. Which of the following must be true? Select all that apply.

    (a) The homogeneous system $Ax = 0$ has only the trivial solution $x = 0$.

    (b) There must be some vector $v$ in $\mathbf{R}^{11}$ so that $Ax = v$ is inconsistent.

    **Solution**: (a) True. There are multiple was to see this. The solution set for any consistent equation $Ax = b$ is a translation of the solution set of $Ax = 0$, so if $Ax = b$ has exactly one solution for some $b$ then $Ax = 0$ must have exactly one solution (namely the trivial solution $x = 0$).

    (b) True: $A$ has 11 rows and 7 columns. Since $A$ can't have more than one pivot in any row or column, we know $A$ has a max of 7 pivots, so it cannot have a pivot in every one of its 11 rows.

17. Compute the product below.

$$\begin{pmatrix} 1 & -2 & 4 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} =$$

    (a) The product is the $2 \times 3$ matrix $\begin{pmatrix} x & -2y & 4z \\ -x & 0 & z \end{pmatrix}$

    (b) $\begin{pmatrix} x - 2y + 4z \\ -x + z \end{pmatrix}$

    (c) $\begin{pmatrix} x - x \\ -2y \\ 4z + z \end{pmatrix}$

    (d) It is not possible to compute the product because the matrix and the vector have incompatible sizes.

    **Solution**: The answer is (b):

$$\begin{pmatrix} 1 & -2 & 4 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x \begin{pmatrix} 1 \\ -1 \end{pmatrix} + y \begin{pmatrix} -2 \\ 0 \end{pmatrix} + z \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} x - 2y + 4z \\ -x + z \end{pmatrix}.$$

18. Suppose that the plane $x_1 - 4x_2 + x_3 = 0$ is the set of solutions to the matrix equation $Ax = 0$, and suppose that $\begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$ is a solution to $Ax = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$.

    (a) Is it true that $A \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$?

    (b) Is it true that $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ is a solution to $Ax = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$?

**Solution**: (a) We could just observe that $\begin{pmatrix}4\\1\\0\end{pmatrix}$ satisfies

$$x_1 - 4x_2 + x_3 = 4 - 4(1) + 0 = 0, \text{ so } \begin{pmatrix}4\\1\\0\end{pmatrix} \text{ is a solution to } Ax = \begin{pmatrix}0\\0\\0\end{pmatrix}.$$

However, this shortcut is not necessary: The plane $x_1 - 4x_2 + x_3 = 0$ corresponds to $\left(\begin{array}{ccc|c}1 & -4 & 1 & 0\end{array}\right)$. This has solution set $x_1 = 4x_2 - x_3$, $x_2 = x_2$, $x_3 = x_3$ (where $x_2$ and $x_3$ are free) and in parametric vector form,

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = x_2\begin{pmatrix}4\\1\\0\end{pmatrix} + x_3\begin{pmatrix}-1\\0\\1\end{pmatrix}.$$

Thus, everything in the Span$\left\{\begin{pmatrix}4\\1\\0\end{pmatrix}, \begin{pmatrix}-1\\0\\1\end{pmatrix}\right\}$ is a solution to $Ax = 0$. In particular $A\begin{pmatrix}4\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$.

(b) Yes. One way to see this is that since $\begin{pmatrix}4\\1\\0\end{pmatrix}$ is a solution to $Ax = \begin{pmatrix}0\\0\\0\end{pmatrix}$ and we were told $\begin{pmatrix}-3\\1\\2\end{pmatrix}$ is a solution to $Ax = \begin{pmatrix}2\\1\\0\end{pmatrix}$, their sum $\begin{pmatrix}4\\1\\0\end{pmatrix} + \begin{pmatrix}-3\\1\\2\end{pmatrix} = \begin{pmatrix}1\\2\\2\end{pmatrix}$ is a solution to $Ax = \begin{pmatrix}2\\1\\0\end{pmatrix}$ by the theory of solution sets. We could also compute this explicitly if we wanted:

$$A\begin{pmatrix}1\\2\\2\end{pmatrix} = A\left(\begin{pmatrix}4\\1\\0\end{pmatrix} + \begin{pmatrix}-3\\1\\2\end{pmatrix}\right) = A\begin{pmatrix}4\\1\\0\end{pmatrix} + A\begin{pmatrix}-3\\1\\2\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} + \begin{pmatrix}2\\1\\0\end{pmatrix} = \begin{pmatrix}2\\1\\0\end{pmatrix}.$$

The matrix $A$ that was actually used for this problem is

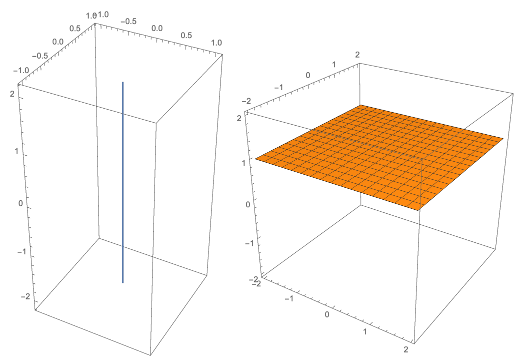
$$A = \begin{pmatrix}-2/5 & 8/5 & -2/5\\ -1/5 & 4/5 & -1/5\\ 0 & 0 & 0\end{pmatrix}.$$

However, it would be incredibly hard to go backwards to reconstruct $A$ from the problem, so we needed to the information we were given in order to answer (a) and (b)!

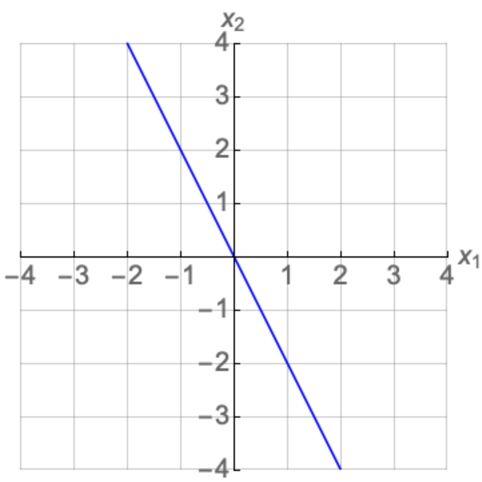
19. Suppose that $A$ is a $3 \times 3$ matrix and the set of solutions to $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ is the $z$-axis in $\mathbf{R}^3$. Is it possible that there is a vector $b$ so that the solutions to $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = b$ is a line in the plane $z = 1$?

**Solution**: No. The $z$-axis is the vertical line given by Span $\left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$, and from the theory of solution sets we know that if $Ax = b$ is consistent then it is a translation of the $z$-axis, which means that the solution set of any consistent equation $Ax = b$ is a (parallel) *vertical* line. However, any line in the plane $z = 1$ is a *horizontal* line.

There was no drawing at all necessary to solve this problem (it just used the relationship between $Ax = 0$ and $Ax = b$), but we draw pictures below of the $z$-axis and the plane $z = 1$ to illustrate.



20. Consider the blue line drawn below, using the convention $(x_1, x_2)$ for points in $\mathbf{R}^2$.




(a) Let $A = \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix}$. Select the correct statement below.

i. The blue line is the set of solutions to $Ax = 0$.
ii. The blue line is the column span of $A$.
iii. The blue line is neither the set of solutions to $Ax = 0$ nor the column span of $A$.

(b) Let $A = \begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix}$. Select the correct statement below.

i. The blue line is the set of solutions to $Ax = 0$.
ii. The blue line is the column span of $A$.
iii. The blue line is neither the set of solutions to $Ax = 0$ nor the column span of $A$.

**Solution**: (a) The answer is (i).

Clearly, the blue line is not the column span of $A$, which is the span of $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$.

The set of solutions to $Ax = 0$ can be found by row-reduction:

$$\left(\begin{array}{cc|c} 2 & 1 & 0 \\ 2 & 1 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right), \qquad 2x_1 + x_2 = 0, x_2 = x_2 \text{ (free)}.$$

This is the line $x_2 = -2x_1$, which is Span $\left\{ \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right\}$.

(b) The answer is (iii).

Clearly, the blue line is not the column span of $A$, which is the span of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. The set of solutions to $Ax = 0$ is given by the line $x_1 - 2x_2 = 0$, which is the line $x_1 = 2x_2$, i.e. Span $\left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$. This is not the blue line.