# Math 1553 Reading Day Fall 2021

ⓘ This is a preview of the published version of the quiz

Started: Nov 30 at 10:25am

# Quiz Instructions

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## Question 1

**1 pts**

If $\{u, v, w\}$ is a set of linearly dependent vectors, then $w$ must be a linear combination of $u$ and $v$.

- ◯ True
- ◯ False

## Question 2

**1 pts**

Find the value of $k$ that makes the following vectors linearly dependent:

$$\begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ -3 \\ k \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$$

## Question 3

**1 pts**

If $\{u, v\}$ is a basis for a subspace $W$, then $\{u - v, u + v\}$ is also a basis for $W$.

○ True

○ False

## Question 4

**1 pts**

Which of the following are subspaces of $\mathbb{R}^4$?

(1) The set $W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbb{R}^4 \quad : \quad 2x - y - z = 0 \right\}$.

(2) The set of solutions to the equation $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 3 & 0 & -1 \end{pmatrix} x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

○ (1) is a subspace but (2) is not a subspace

○ neither is a subspace

○ (2) is a subspace but (1) is not a subspace

○ both are subspaces

## Question 5

**1 pts**

Let $W$ be the set of vectors $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ in $\mathbb{R}^3$ with $abc = 0$. Then $W$ is closed under addition, meaning that if $v$ and $w$ are in $W$, then $v + w$ is in $W$.

○ True

○ False

## Question 6

1 pts

Match the transformations given below with their corresponding $2 \times 2$ matrix.

A. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

B. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

C. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

D. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

E. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

| | |
|---|---|
| Counter-clockwise rotation by 90 degrees | [ Choose ] |
| Reflection about the line y=x | [ Choose ] |
| Clockwise rotation by 90 degrees | [ Choose ] |
| Reflection across the x-axis | [ Choose ] |
| Reflection across the y-axis | [ Choose ] |

## Question 7

1 pts

Find the value of $k$ so that the matrix transformation for the following matrix is not onto.

$$\begin{pmatrix} 1 & 3 & 9 \\ 2 & 6 & k \end{pmatrix}$$

## Question 8

**1 pts**

Find the **nonzero** value of $k$ that makes the following matrix not invertible.

$$\begin{pmatrix} 1 & -1 & 0 \\ k & k^2 & 0 \\ -1 & 1 & 5 \end{pmatrix}$$

*Enter an integer as your answer. Note that 0 is not the correct answer, since the question asks for a nonzero value of $k$.*

## Question 9

**1 pts**

Match the following definitions with the corresponding term describing a linear transformation $T : \mathbb{R}^m \to \mathbb{R}^n$.

Each definition should be used exactly once.

A. For each $y$ in $\mathbb{R}^n$ there is at most one $x$ in $\mathbb{R}^m$ so that $T(x) = y$.

B. For each $y$ in $\mathbb{R}^n$ there is at least one $x$ in $\mathbb{R}^m$ so that $T(x) = y$.

C. For each $y$ in $\mathbb{R}^n$ there is exactly one $x$ in $\mathbb{R}^m$ so that $T(x) = y$.

D. For each $x$ in $\mathbb{R}^m$ there is exactly one $y$ in $\mathbb{R}^n$ so that $T(x) = y$.

T is a transformation [ Choose ]

| | |
|---|---|
| T is one-to-one | [ Choose ] |
| T is onto | [ Choose ] |
| T is one-to-one and onto | [ Choose ] |

## Question 10

**1 pts**

Suppose $A$ is a $4 \times 6$ matrix. Then the dimension of the null space of $A$ is at most 2.

- True
- False

## Question 11

**1 pts**

Complete the entries of the matrix $A$ so that $\mathrm{Col}(A) = \mathrm{Span}\left\{\begin{pmatrix}1\\2\end{pmatrix}\right\}$ and $\mathrm{Nul}(A) = \mathrm{Span}\left\{\begin{pmatrix}1\\1\end{pmatrix}\right\}$.

$A = \begin{pmatrix} r & 1 \\ s & 2 \end{pmatrix}$, where $r$ = [ ] and $s$ = [ ]

## Question 12

**1 pts**

Suppose $T : \mathbb{R}^7 \to \mathbb{R}^9$ is a linear transformation with standard matrix $A$, and suppose that the range of $T$ has a basis consisting of 3 vectors. What is the dimension of the null space of $A$?

## Question 13

1 pts

Define a transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ by $T(x, y, z) = (0,\ x - y,\ y - x,\ z)$.

Which *one* of the following statements is true?

- $T$ is onto but not one-to-one.
- $T$ is one-to-one and onto.
- $T$ is neither one-to-one nor onto.
- $T$ is one-to-one but not onto.

## Question 14

1 pts

Suppose that $A$ is a $7 \times 5$ matrix, and the null space of $A$ is a line. Say that $T$ is the matrix transformation $T(v) = Av$. Which of the following statements must be true about the range of $T$?

- It is a 4-dimensional subspace of $\mathbb{R}^7$
- It is a 4-dimensional subspace of $\mathbb{R}^5$
- It is a 6-dimensional subspace of $\mathbb{R}^5$
- It is a 6-dimensional subspace of $\mathbb{R}^7$

## Question 15

1 pts

Say that $S : \mathbb{R}^2 \to \mathbb{R}^3$ and $T : \mathbb{R}^3 \to \mathbb{R}^4$ are linear transformations. Which of the following must be true about $T \circ S$?

- ◯ It is one-to-one
- ◯ The composition is not defined
- ◯ It is onto
- ◯ It is not onto
- ◯ It is not one-to-one

## Question 16

**1 pts**

Suppose that $A$ is an invertible $n \times n$ matrix. Then $A + A$ must be invertible.

- ◯ True
- ◯ False

## Question 17

**1 pts**

Suppose $A$ is a $3 \times 3$ matrix and the equation $Ax = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$ has exactly one solution.

Then $A$ must be invertible.

- ◯ True
- ◯ False

## Question 18

1 pts

Suppose that $A$ and $B$ are $n \times n$ matrices and $AB$ is not invertible.

Which *one* of of the following statements must be true?

- None of these
- A is not invertible
- B is not invertible
- At least one of the matrices A or B is not invertible

## Question 19

1 pts

Suppose $A$ and $B$ are $3 \times 3$ matrices, with $\det(A) = 3$ and $\det(B) = -6$.

Find $\det(2A^{-1}B)$.

## Question 20

1 pts

Let $A$ be the $3 \times 3$ matrix satisfying $Ae_1 = e_3,\ Ae_2 = e_2$, and $Ae_3 = 2e_1$ (recall that we use $e_1$, $e_2$, and $e_3$ to denote the standard basis vectors for $\mathbb{R}^3$).

Find $\det(A)$.

## Question 21

1 pts

Suppose $A$ is a square matrix and $\lambda = -1$ is an eigenvalue of $A$.

Which one of the following statements must be true?

- $A$ is invertible.
- The equation \(Ax = x \\)has only the trivial solution.
- $\text{Nul}(A + I) = \{0\}$
- For some nonzero $x$, the vectors $Ax$ and $x$ are linearly dependent.
- The columns of $A + I$ are linearly independent.

## Question 22

1 pts

Suppose $A$ is a 4 x 4 matrix with characteristic polynomial $-(1-\lambda)^2(5-\lambda)\lambda$.

What is the rank of $A$?

## Question 23

1 pts

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that reflects across the line $x_2 = 2x_1$.

Find the value of $k$ so that $A \begin{pmatrix} 2 \\ k \end{pmatrix} = \begin{pmatrix} 2 \\ k \end{pmatrix}$.

## Question 24

1 pts

Find the value of $k$ such that the matrix $\begin{pmatrix} 1 & k \\ 1 & 3 \end{pmatrix}$ has one real eigenvalue of algebraic multiplicity 2. *Enter an integer value below.*

## Question 25

**1 pts**

Suppose that $A$ is a $5 \times 5$ matrix with characteristic polynomial $(1-\lambda)^3(2-\lambda)(3-\lambda)$ and also that $A$ is diagonalizable. What is the dimension of the 1-eigenspace of $A$?

## Question 26

**1 pts**

Find the value of $t$ such that 3 is an eigenvalue of $\begin{pmatrix} 1 & t & 3 \\ 1 & 1 & 1 \\ 3 & 0 & 3 \end{pmatrix}$. *Enter an integer answer below.*

## Question 27

**1 pts**

Say that $A$ is a $2 \times 2$ matrix with characteristic polynomial $(1 - \lambda)(2 - \lambda)$. What is the characteristic polynomial of $A^2$?

- ○ $(1 - \lambda)^2(2 - \lambda)^2$
- ○ $(1 - \lambda)(4 - \lambda)$
- ○ $(1 - \lambda^2)(2 - \lambda^2)$
- ○ $(1 - \lambda)(2 - \lambda)$
- ○ $(1 - \lambda^2)(4 - \lambda^2)$

## Question 28

**1 pts**

Suppose that a vector $x$ is an eigenvector of $A$ with eigenvalue 3 and that $x$ is also an eigenvector of $B$ with eigenvalue 4. Which of the following is true about the matrix $2A - B$ and $x$:

- ○ $x$ is an eigenvector of $2A - B$ with eigenvalue 3
- ○ $x$ is an eigenvector of $2A - B$ with eigenvalue 1
- ○ $x$ is an eigenvector of $2A - B$ with eigenvalue 2
- ○ None of these
- ○ $x$ is an eigenvector of $2A - B$ with eigenvalue 4

## Question 29

**1 pts**

Suppose that $A$ is a $4 \times 4$ matrix with eigenvalues 0, 1, and 2, where the eigenvalue 1 has algebraic multiplicity two.

Which of the following must be true?

(1) $A$ is not diagonalizable

(2) $A$ is not invertible

- ○ Both (1) and (2) must be true
- ○ Neither statement is necessarily true
- ○ (2) must be true but (1) might not be true
- ○ (1) must be true but (2) might not be true

## Question 30

**1 pts**

Suppose $A$ is a $5 \times 5$ matrix whose entries are real numbers. Then $A$ must have at least one real eigenvalue.

- ○ True
- ○ False

## Question 31

**1 pts**

Suppose $A$ is a positive stochastic matrix and $A\begin{pmatrix} 3/5 \\ 2/5 \end{pmatrix} = \begin{pmatrix} 3/5 \\ 2/5 \end{pmatrix}$. Let $v = \begin{pmatrix} 5 \\ 95 \end{pmatrix}$.

As $n$ gets very large, $A^n v$ approaches the vector $\begin{pmatrix} r \\ s \end{pmatrix}$, where:

$r$ = [ ] and $s$ = [ ] .

## Question 32

**1 pts**

Suppose that $A$ is a $4 \times 4$ matrix of rank 2. Which one of the following statements must be true?

- $A$ must have four distinct eigenvalues
- $A$ is not diagonalizable
- none of these
- $A$ cannot have four distinct eigenvalues
- $A$ is diagonalizable

## Question 33

1 pts

Suppose $A$ is a $2 \times 2$ matrix whose entries are real numbers, and suppose $A$ has eigenvalue $1 + i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1+i \end{pmatrix}$.

Which of the following must be true?

- $A$ must have eigenvalue $1 - i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1+i \end{pmatrix}$
- None of these
- $A$ must have eigenvalue $1 + i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1-i \end{pmatrix}$
- $A$ must have eigenvalue $1 - i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1-i \end{pmatrix}$

## Question 34

1 pts

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that rotates the plane clockwise by 45 degrees, and let $A$ be the standard matrix for $T$.

Which *one* of the following statements is true?

- ◯ $A$ has one complex eigenvalue with algebraic multiplicity two
- ◯ $A$ has one real eigenvalue with algebraic multiplicity two
- ◯ $A$ has two distinct complex eigenvalues.
- ◯ $A$ has two distinct real eigenvalues

## Question 35

**1 pts**

Suppose $u$ and $v$ are orthogonal unit vectors (to say that a vector is a unit vector means that it has length 1). Find the dot product

$(3u - 8v) \cdot 4u.$

## Question 36

**1 pts**

Find the value of $k$ that makes the following pair of vectors orthogonal.

$$\begin{pmatrix} 2 \\ k \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} k \\ 1 \\ -6 \end{pmatrix}$$

*Your answer should be an integer.*

## Question 37

**1 pts**

If $W$ is a subspace of $\mathbb{R}^{100}$ and $v$ is a vector in $W^{\perp}$ then the orthogonal projection of $v$ to $W$ must be the $0$ vector.

- True
- False

## Question 38 1 pts

Suppose $W$ is a subspace of $\mathbb{R}^n$. If $x$ is a vector and $x_W$ is the orthogonal projection of $x$ onto $W$, then $x \cdot x_W$ must be 0.

- True
- False

## Question 39 1 pts

Suppose that $A$ is a $3 \times 3$ invertible matrix. What is the dot product between the second row of $A$ and third column of $A^{-1}$ equal to?

- 1
- 0
- Not Enough Information is Given
- -1
- -2
- 2

## Question 40 1 pts

Find the orthogonal projection of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ onto $\mathrm{Span}\left\{\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right\}$.

The orthogonal projection is $\begin{pmatrix} a \\ b \end{pmatrix}$, where: $a$ = ______ and $b$ = ______ .

*Enter integers or fractions as your entries.*

## Question 41

**1 pts**

Compute the orthogonal projection of the vector $\begin{pmatrix} 6 \\ 5 \\ 4 \end{pmatrix}$ to the plane spanned by the vectors $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$. What is the first coordinate of the projection? *Your answer should be an integer.*

## Question 42

**1 pts**

Suppose $B$ is the standard matrix for the transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ of orthogonal projection onto the subspace $W = \left\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ in } \mathbb{R}^3 \;\middle|\; x + y + 2z = 0\right\}$.

What is the dimension of the 1-eigenspace of $B$?

## Question 43

**1 pts**

Let $W$ be the subspace of $\mathbb{R}^4$ given by all vectors $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$ such that $x - y + z + w = 0$. Find dimension of the orthogonal complement $W^\perp$.

## Question 44

**1 pts**

If $b$ is in the column space of the matrix $A$ then every solution to $Ax = b$ is a least squares solution.

- ○ True
- ○ False

## Question 45

**1 pts**

If $A$ is an $m \times n$ matrix, $b$ is in $\mathbb{R}^m$, and $\hat{x}$ is a least squares solution to $Ax = b$, then $\hat{x}$ is the point in $\mathrm{Col}(A)$ that is closest to $b$.

- ○ True
- ○ False

## Question 46

1 pts

Find the least squares solution $\hat{x}$ to the linear system

$$\begin{pmatrix} 6 \\ -2 \\ -2 \end{pmatrix} x = \begin{pmatrix} 14 \\ -2 \\ 0 \end{pmatrix}.$$

If your answer is an integer, enter an integer.

If your answer is not an integer, enter a fraction.

## Question 47

1 pts

Find the best fit line y= [ ] x+ [ ] for the data points $(-7, -22), (0, -2),$ and $(7, 6)$ using the method of least squares. *Your answers should both be integers.*

## Question 48

1 pts

Let $A = \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} -3 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix}^{-1}.$

Find $r$ and $s$ so that $A^3 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} r \\ s \end{pmatrix}.$

$r$ =

$s$ =

## Question 49

1 pts

If $A$ is a diagonalizable $6 \times 6$ matrix, then $A$ has $6$ distinct eigenvalues.

- ◯ True
- ◯ False

## Question 50

1 pts

Find the eigenvalues of the matrix $A = \begin{pmatrix} 1 & 4 \\ 4 & 7 \end{pmatrix}$ and write them in increasing order.

The smaller eigenvalue is $\lambda_1$ = [ ] .

The larger eigenvalue is $\lambda_2$ = [ ] .

No new data to save. Last checked at 10:25am

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