Math 1553 Worksheet §5.6 - §6.5

1. Courage Soda and Dexter Soda compete for a market of 210 customers who drink soda each day.
Today, Courage has 80 customers and Dexter has 130 customers. Each day:
70% of Courage Soda's customers keep drinking Courage Soda, while 30% switch to Dexter Soda.
40% of Dexter Soda's customers keep drinking Dexter Soda, while 60% switch to Courage Soda.

a) Write a stochastic matrix A and a vector x so that Ax will give the number of customers for Courage Soda and Dexter Soda (in that order) tomorrow. You do not need to compute Ax.

b) Find the steady-state vector for A.

c) Use your answer from (b) to determine the following: in the long run, roughly how many daily customers will Courage Soda have?

2. True/False

(1) If u is in subspace W, and u is also in $W^{\perp}$, then $u = 0$.

(2) If y is in a subspace W, the orthogonal projection of y onto $W^{\perp}$ is 0.

(3) If x is orthogonal to v and w, then x is also orthogonal to $v - w$.

3. **a)** Find the standard matrix B for proj_L, where $L = \text{Span}\left\{\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}\right\}$.

b) What are the eigenvalues of B? Is B is diagonalizable?

4. $y = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}, \quad u_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad u_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

(1) Determine whether u_1 and u_2
   (a) are linearly independent
   (b) are orthogonal
   (c) span $\mathbf{R}^3$

(2) Is y in $W = \text{Span}\{u_1, u_2\}$?

(3) Compute the vector w that most closely approximates y within W.

(4) Construct a vector, z, that is in $W^\perp$.

(5) Make a rough sketch of W, y, w, and z.

5. Find the best fit line $y = Ax + B$ through the points $(0, 0)$, $(1, 8)$, $(3, 8)$, and $(4, 20)$.