

Math 1553 Worksheet §3.4-3.6

1. True or false. Answer true if the statement is *always* true. Otherwise, answer false. If your answer is false, either give an example that shows it is false or (in the case of an incorrect formula) state the correct formula.
- a) If A and B are $n \times n$ matrices and both are invertible, then the inverse of AB is $A^{-1}B^{-1}$.
- b) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in \mathbf{R}^n , then the solution is *unique* for each b in \mathbf{R}^n .
- c) If A is an $n \times n$ matrix and the equation $Ax = b$ has at most one solution for each b in \mathbf{R}^n , then the solution must be *unique* for each b in \mathbf{R}^n .
- d) If A and B are invertible $n \times n$ matrices, then $A+B$ is invertible and $(A+B)^{-1} = A^{-1} + B^{-1}$.
- e) If A is a 3×4 matrix and B is a 4×2 matrix, then the linear transformation Z defined by $Z(x) = ABx$ has domain \mathbf{R}^3 and codomain \mathbf{R}^2 .

2. A is $m \times n$ matrix, B is $n \times m$ matrix. Select all correct answers from the box. It is possible to have more than one correct answer.

a) Suppose x is in \mathbf{R}^m . Then ABx must be in:

$\text{Col}(A)$, $\text{Nul}(A)$, $\text{Col}(B)$, $\text{Nul}(B)$

b) Suppose x in \mathbf{R}^n . Then BAx must be in:

$\text{Col}(A)$, $\text{Nul}(A)$, $\text{Col}(B)$, $\text{Nul}(B)$

c) If $m > n$, then columns of AB could be linearly *independent*, *dependent*

d) If $m > n$, then columns of BA could be linearly *independent*, *dependent*

e) If $m > n$ and $Ax = 0$ has nontrivial solutions, then columns of BA could be linearly *independent*, *dependent*

3. Consider the following linear transformations:

$T: \mathbf{R}^3 \rightarrow \mathbf{R}^2$ T projects onto the xy -plane, forgetting the z -coordinate

$U: \mathbf{R}^2 \rightarrow \mathbf{R}^2$ U rotates clockwise by 90°

$V: \mathbf{R}^2 \rightarrow \mathbf{R}^2$ V scales the x -direction by a factor of 2.

Let A, B, C be the matrices for T, U, V , respectively.

a) Write A, B , and C .

b) Compute the matrix for $V \circ U \circ T$.

c) Compute the matrix for $U \circ V \circ T$.

d) Describe U^{-1} and V^{-1} , and compute their matrices.