MATH 1553, FALL 2023
SAMPLE MIDTERM 2A: COVERS 2.5 - 3.4

Name		**GT ID**	

Please **read all instructions** carefully before beginning.

- Write your initials at the top of each page.
- The maximum score on this exam is 70 points, and you have 75 minutes to complete this exam. Each problem is worth 10 points.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means “reduced row echelon form.”
- The “zero vector” in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and all work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have more than enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam*. If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is meant to be similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. We recommend completing the practice exam in 75 minutes, without notes or distractions.

The exam is not designed to test material from the previous midterm on its own. However, knowledge of the material prior to section §2.5 is necessary for everything we do for the rest of the semester, so it is fair game for the exam as it applies to §§2.5 through 3.4.

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Problem 1.

For each statement, answer TRUE or FALSE. If the statement is *ever* false, circle FALSE. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

Answers are given below. Solutions are given on the next page.

a) Suppose v_1, v_2, and v_3 are linearly dependent vectors in $\mathbf{R}^4$. Then v_1 must be a linear combination of v_2 and v_3.

TRUE **[FALSE]**

b) If A is a 3×8 matrix, then $\dim(\text{Nul}\,A) > \dim(\text{Col}\,A)$.

[TRUE] FALSE

c) Consider the subspace W of $\mathbf{R}^4$ given by

$$W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; x - y - z + w = 0 \right\}.$$

Then $\dim(W) = 3$.

[TRUE] FALSE

d) Suppose $T : \mathbf{R}^4 \to \mathbf{R}^3$ is a transformation. Then for each y in $\mathbf{R}^3$, there is a vector x in $\mathbf{R}^4$ so that $T(x) = y$.

TRUE **[FALSE]**

e) There is a 3×4 matrix whose null space is a plane and whose column space is a line.

TRUE **[FALSE]**

Solution.

a) **False**. Similar to many past examples including an example in the 2.5 slides. Out of pure coincidence, it is nearly identical to #5 in the 2.5-3.1 Supplement. If the set is linearly dependent, we only know that *at least one* of the vectors is a linear combination of the others, not that *every* vector is. For example, the vectors below form a linearly dependent set but v_1 is not a linear combination of v_2 and v_3.

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}.$$

b) **True**. We could count pivots or use the Rank Theorem. Either way, we see that there are 8 columns in A, so there are most 3 pivots and the equation $Ax = 0$ will have at least 5 free variables in its solution sets.

This means $\dim(\text{Nul } A) \geq 5$ and $\dim(\text{Col } A) \leq 3$.

c) **True**. $W = \text{Nul}(A)$ for $A = \begin{pmatrix} 1 & -1 & -1 & 1 \end{pmatrix}$. We see A has 1 pivot and 4 columns, so $\dim(W) = \dim(\text{Nul } A) = 3$. In fact, we saw that $\dim(W) = 3$ way back in section 1.1 at the beginning of the semester!

This problem is similar to an example in the 2.7+2.9 notes as well as the 2.6 Supplement #1c and #6.

d) **False**. This was essentially taken from #1 on the 3.2 Webwork and nearly the same as Quiz 5 #2. The statement is false because it is not necessarily true for a general transformation (it would be true if we added the assumption that T was an **onto** transformation). For example, if T is the transformation

$$T(x_1, x_2, x_3, x_4) = (x_1, 0, 0)$$

then for $y = (0, 1, 0)$ there is no x in $\mathbf{R}^4$ so that $T(x) = y$.

e) **False**. By the Rank Theorem, the dimensions of the null space of A and column space of A must add to exactly 4. In the proposed scenario, the dimensions would add to 3.

Problem 2.

Parts (a), (b), and (c) are unrelated. There is no work required and no partial credit on this page.

a) (4 points) In each case, clearly circle YES or NO.

(i) Let V be the set of all vectors of the form $\begin{pmatrix} x \\ 0 \end{pmatrix}$ in $\mathbf{R}^2$, where x is any real number.

Is V a subspace of $\mathbf{R}^2$? $\boxed{\text{YES}}$ NO

(ii) Let W be the set in $\mathbf{R}^3$ consisting of all solutions to the vector equation

$$x_1 \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

Is W a subspace of $\mathbf{R}^3$? YES $\boxed{\text{NO}}$

(iii) Suppose A is a 3×3 matrix. Must it be true that the solution set of the matrix equation $Ax = 0$ is a subspace of $\mathbf{R}^3$? $\boxed{\text{YES}}$ NO

(iv) Suppose $T : \mathbf{R}^2 \to \mathbf{R}^4$ is a linear transformation. Must it be true that the range of T is a subspace of $\mathbf{R}^4$? $\boxed{\text{YES}}$ NO

b) (3 points) Suppose $\{v_1, v_2, v_3, v_4\}$ is a **linearly independent** set of vectors in $\mathbf{R}^4$. Which of the following statements are true? Clearly circle all that apply.

$\boxed{\text{(i)}}$ For each b in $\mathbf{R}^4$, the vector equation

$$x_1 v_1 + x_2 v_2 + x_3 v_3 + x_4 v_4 = b$$

is consistent and has a unique solution.

(ii) It is possible that the set $\{v_1, v_2, v_3\}$ is linearly dependent.

$\boxed{\text{(iii)}}$ $\text{Span}\{v_1, v_2, v_3, v_4\} = \mathbf{R}^4$.

c) (3 points) Suppose $\{u, v, w\}$ is a basis for some subspace V of $\mathbf{R}^n$. Which of the following must be true? Clearly circle all that apply.

$\boxed{\text{(i)}}$ If $\{a, b, c\}$ are vectors in V and $\text{Span}\{a, b, c\} = V$, then $\{a, b, c\}$ must be a basis for V.

$\boxed{\text{(ii)}}$ The set $\{u,\ u + 2v,\ v + w\}$ must be a basis for V.

$\boxed{\text{(iii)}}$ If $\{a, b, c\}$ is any set of 3 linearly independent vectors in V, then $\{a, b, c\}$ must be a basis for V.

Solution.

a) (i) Yes, since $V = \text{Span}\left\{\begin{pmatrix}1\\0\end{pmatrix}\right\}$, i.e. V is the x-axis.

(ii) No: W is the solution set to $\begin{pmatrix}2 & 3 & 0\\ -1 & 2 & 0\\ 0 & 0 & 1\end{pmatrix}x = \begin{pmatrix}0\\1\\1\end{pmatrix}$, so it does not include the zero vector.

(iii) Yes, the null space of any 3×3 matrix is a subspace of $\mathbf{R}^3$.

(iv) Yes, the range of T is the column space of its standard (4×2) matrix A, which is a subspace of $\mathbf{R}^4$. This part was taken from Quiz 5.

b) (i) Yes: by linear independence of the vectors, the 4×4 matrix A whose columns are v_1 through v_4 has 4 pivots, thus a pivot in each row and column. This guarantees any such vector equation is consistent (by pivot in each row) and has a unique solution (by pivot in each column).

(ii) No: If $\{v_1, v_2, v_3\}$ were linearly dependent then automatically $\{v_1, v_2, v_3, v_4\}$ would be linearly dependent.

(iii) Yes: by the same reasoning as (i), A has a pivot in each row, therefore its column span (the span of v_1 through v_4) is all of $\mathbf{R}^4$.

c) (i) Yes, since $\dim(V) = 3$, any 3 vectors in V that span V form a basis of V by the Basis Theorem.

(ii) Yes, the set $\{u, u + 2v, v + w\}$ is a linearly independent set of vectors in V by the increasing span criterion, therefore it is a basis for the (three-dimensional) subspace V by the Basis Theorem.

(iii) Yes: since $\dim(V) = 3$, any 3 linearly independent vectors in V form a basis of V by the Basis Theorem.

Problem 3.

Parts (a), (b), and (c) are unrelated. You do not need to show your work, and there is no partial credit.

a) (3 points) Consider the set $V = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \text{ in } \mathbf{R}^2 \;\middle|\; x = y \right\}$.

(i) Is V closed under addition? $\boxed{\text{YES}}$ NO

(ii) Is V closed under scalar multiplication? $\boxed{\text{YES}}$ NO

(iii) Is there a matrix A so that $\mathrm{Col}(A) = V$? $\boxed{\text{YES}}$ NO

b) (4 points) Suppose A is a 3×4 matrix and B is a 4×5 matrix, and let T be the matrix transformation $T(x) = ABx$. Which of the following must be true? Clearly circle all that apply.

(i) The null space of AB is a subspace of $\mathbf{R}^4$.

$\boxed{\text{(ii)}}$ Every vector in the column space of AB is also in the column space of A.

$\boxed{\text{(iii)}}$ T cannot be one-to-one.

c) (3 points) Which of the following transformations are linear? Clearly circle all that apply.

(i) $T : \mathbf{R}^2 \to \mathbf{R}^2$ given by $T(x_1, x_2) = (x_1, |x_2|)$.

$\boxed{\text{(ii)}}$ $T : \mathbf{R}^3 \to \mathbf{R}^3$ given by $T(x_1, x_2, x_3) = (x_1 - x_2, x_3, x_1)$.

$\boxed{\text{(iii)}}$ $T : \mathbf{R}^2 \to \mathbf{R}^2$ that reflects vectors across the line $y = -x$.

Solution.

a) The answer is yes to all of (i), (ii), and (iii). V is the span of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ so V is a subspace of $\mathbf{R}^2$, therefore it is closed under addition and scalar multiplication. Furthermore, V is the column span of (for example) the matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, but there are also many other matrices A so that $\mathrm{Col}(A) = V$.

b) Note that the matrix AB is 3×5 and therefore the transformation $T(x) = ABx$ has domain $\mathbf{R}^5$ and codomain $\mathbf{R}^3$.
(i) No. The null space of AB is a subspace of $\mathbf{R}^5$ since AB is 3×5.
(ii) Yes.
(iii) Yes: since AB has 5 columns but a max of 3 pivots, the columns of AB must be linearly dependent, therefore T cannot be one-to-one.

c) This is a slight modification of #4 in the 3.3 Webwork.
(i) No, the absolute value ruins it.
(ii) Yes, it is linear.
(iii) Yes, this is the matrix transformation $T(x) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} x$.

Problem 4.

You do not need to show your work on this problem, and there is no partial credit. Parts (a), (b), and (c) are unrelated.

a) (3 points) In each case, consider the matrix transformation $T(x) = Ax$. Determine whether T is one-to-one and whether T is onto. If T is one-to-one, clearly circle "one-to-one." If T is onto, clearly circle "onto." If T is neither one-to-one nor onto, do not circle anything. If T is one-to-one and onto, circle one-to-one and circle onto.

(I) $A = \begin{pmatrix} \cos(\pi/10) & -\sin(\pi/10) \\ \sin(\pi/10) & \cos(\pi/10) \end{pmatrix}$ $\quad$ $\boxed{\text{one-to-one}}$ $\quad$ $\boxed{\text{onto}}$

(II) $A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 0 & 2 \end{pmatrix}$ $\quad$ one-to-one $\quad$ $\boxed{\text{onto}}$

(III) $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$ $\quad$ $\boxed{\text{one-to-one}}$ $\quad$ onto

b) (4 points) Let $A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix}$, and let T be the matrix transformation $T(x) = Ax$.

(I) What is the domain of T? Clearly circle your answer below.

$\mathbf{R}$ $\quad$ $\mathbf{R}^2$ $\quad$ $\boxed{\mathbf{R}^3}$ $\quad$ $\mathbf{R}^4$ $\quad$ $\mathbf{R}^5$

(II) What is the codomain of T? Clearly circle your answer below.

$\mathbf{R}$ $\quad$ $\boxed{\mathbf{R}^2}$ $\quad$ $\mathbf{R}^3$ $\quad$ $\mathbf{R}^4$ $\quad$ $\mathbf{R}^5$

(III) What is the null space of A? Clearly circle your answer below.

a point in $\mathbf{R}^2$ $\quad$ a line in $\mathbf{R}^2$ $\quad$ a point in $\mathbf{R}^3$ $\quad$ $\boxed{\text{a line in } \mathbf{R}^3}$ $\quad$ a plane in $\mathbf{R}^3$

(IV) Is T onto? Clearly circle your answer below.

$\boxed{\text{YES}}$ $\quad$ NO

c) (3 points) Consider the transformation

$$T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

(i) Write a matrix A so that $T(x) = Ax$.

(ii) Is there a set of three linearly independent vectors in the range of T?

Solution.

a) (I) Rotation by $\pi/10$ radians (or any angle) counterclockwise about the origin is one-to-one and onto.
(II) A has a pivot in each row but not each column, so T is onto but not one-to-one.
(III) A has a pivot in each column but not each row, so T is one-to-one but not onto.

b) Parts (I) and (II) are fundamental and were essentially taken from #3 in the 3.1 Webwork. Part (III) is a concept check for the null space of the matrix, and part (IV) is a standard question about onto transformations.
(I) and (II): A is a 2×3 matrix, so the domain of T is $\mathbf{R}^3$ and the codomain is $\mathbf{R}^2$.
(III) A has two pivots but three columns, so the solution set to $Ax = 0$ is a line in $\mathbf{R}^3$.
(IV) T is onto because A has a pivot in each row.

c) (i) $A = \begin{pmatrix} T(e_1) & T(e_2) \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 3 & 1 \end{pmatrix}$.

(ii) No: the range of T is spanned by $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$, so there cannot be a set of three linearly independent vectors in the range of T.

Problem 5.

Free response. Show your work unless otherwise indicated! A correct answer without sufficient work may receive little or no credit.

Consider the matrix A and its reduced row echelon form given below.

$$A = \begin{pmatrix} -4 & 4 & -8 & -13 \\ 3 & -3 & 6 & 10 \\ -5 & 5 & -10 & -16 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

a) (2 points) Write a basis for Col A. There is no work required on this part.

b) (4 points) Find a basis for Nul A.

c) (2 points) Write one nonzero vector in the null space of A. There is no work required and no partial credit for this part.

d) (2 pts) Let T be the matrix transformation $T(x) = Ax$. Are there two different vectors u and v (with $u \neq v$) satisfying $T(u) = T(v)$?
If your answer is yes, write such vectors u and v. If your answer is no, justify why not.

Solution.

a) The pivot columns of A will form a basis for Col(A): $\left\{ \begin{pmatrix} -4 \\ 3 \\ -5 \end{pmatrix}, \begin{pmatrix} -13 \\ 10 \\ -16 \end{pmatrix} \right\}$. However, in this problem, any two columns of A will form a basis for Col(A) as long as one of the two columns chosen is $\begin{pmatrix} -13 \\ 10 \\ -16 \end{pmatrix}$.

b) From the RREF of A, for the solution set for $Ax = 0$ we see x_2 and x_3 are free and

$$x_1 - x_2 + 2x_3 = 0, \qquad x_4 = 0.$$

Therefore, $x_1 = x_2 - 2x_3$ and

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_2 - 2x_3 \\ x_2 \\ x_3 \\ 0 \end{pmatrix} = x_2 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -2 \\ 0 \\ 1 \\ 0 \end{pmatrix}. \qquad \text{Basis for Nul}(A): \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}.$$

c) Any **nonzero** linear combination of $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ is correct.

d) Many answers possible. One easy way to do this is to choose two different vectors in the nullspace. For example, $x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ and $y = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$.

Problem 6.

Free response. Show your work except in part (c). A correct answer without sufficient work may receive little or no credit. In this problem:

$T : \mathbf{R}^3 \to \mathbf{R}^2$ is the linear transformation $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x - y + 2z \\ z - x \end{pmatrix}$.

$U : \mathbf{R}^2 \to \mathbf{R}^2$ is the linear transformation that rotates vectors **clockwise** by 45 degrees.
$V : \mathbf{R}^2 \to \mathbf{R}^2$ is the linear transformation that reflects vectors across the line $y = x$.

a) (3 points) Find the standard matrix A for T.

b) (2 points) Write the standard matrix B for U.
(do *not* leave your answer in terms of sine and cosine; simplify it completely)

c) (2 points) Write the standard matrix C for V.

d) (3 pts) Let $W : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation that first reflects vectors across the line $y = x$, then rotates by 45 degrees clockwise. Find the standard matrix D for W.

Solution.

a) $A = \begin{pmatrix} T(e_1) & T(e_2) & T(e_3) \end{pmatrix} = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 0 & 1 \end{pmatrix}$

b) This appeared on the sample exam and is a standard formula for clockwise rotations.

$$B = \begin{pmatrix} \cos(\pi/4) & \sin(\pi/4) \\ -\sin(\pi/4) & \cos(\pi/4) \end{pmatrix},$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \text{ or equivalently } \left(\text{since } \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}\right) B = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}.$$

c) This is one of our most common matrix transformations.

$$C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

d) $W = U \circ V$ since we first do V then do U, so the matrix is $D = BC$.

$$D = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

Alternatively, we can do the problem geometrically, since

$$D = \left(W\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad W\begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$$

where:
(1) Reflecting $\begin{pmatrix}1\\0\end{pmatrix}$ across $y = x$ gives us $\begin{pmatrix}0\\1\end{pmatrix}$, whereby rotating 45° clockwise gives

$$W\begin{pmatrix}1\\ \\ 0\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \\ \frac{1}{\sqrt{2}}\end{pmatrix}.$$

(2) Reflecting $\begin{pmatrix}0\\1\end{pmatrix}$ across $y = x$ gives us $\begin{pmatrix}1\\0\end{pmatrix}$, whereby rotating 45° clockwise gives

$$W\begin{pmatrix}0\\ \\ 1\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \\ -\frac{1}{\sqrt{2}}\end{pmatrix}.$$

Problem 7.

Free response. Show your work! A correct answer without sufficient work may receive little or no credit. Parts (a), (b), and (c) are unrelated.

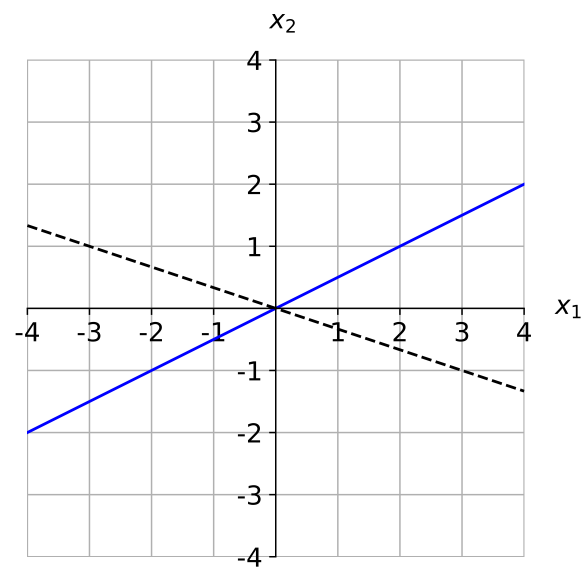
Solutions are given on the next page.

a) (3 points) Suppose $T : \mathbf{R}^3 \to \mathbf{R}^2$ is a linear transformation satisfying

$$T\begin{pmatrix}-1\\1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} \quad \text{and} \quad T\begin{pmatrix}1\\1\\-1\end{pmatrix} = \begin{pmatrix}-1\\2\end{pmatrix}.$$

Find $T\begin{pmatrix}-2\\0\\1\end{pmatrix}$.

b) (4 points) Find a matrix A whose **column space** is the **dotted** line below and whose null space is the solid diagonal line below.



c) (3 points) Let A be the matrix that reflects vectors counterclockwise by 10 degrees. Find $A^9\begin{pmatrix}2\\0\end{pmatrix}$.

Solution.

a) Similar to the 3.3 Webwork #3. $\begin{pmatrix}-2\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\1\\0\end{pmatrix} - \begin{pmatrix}1\\1\\-1\end{pmatrix}$, so by the linearity property $T(u-v) = T(u) - T(v)$:

$$T\begin{pmatrix}-2\\0\\1\end{pmatrix} = T\begin{pmatrix}-1\\1\\0\end{pmatrix} - T\begin{pmatrix}1\\1\\-1\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix}.$$

b) We need A to be a 2×2 matrix whose column span is $\text{Span}\left\{\begin{pmatrix}3\\-1\end{pmatrix}\right\}$ and null space is $\text{Span}\left\{\begin{pmatrix}2\\1\end{pmatrix}\right\}$.

Nul(A) corresponds to the parametric vector form $\begin{pmatrix}x_1\\x_2\end{pmatrix} = x_2\begin{pmatrix}2\\1\end{pmatrix}$, thus $x_1 = 2x_2$ or equivalently $x_1 - 2x_2 = 0$, so we want the second column to be -2 times the first. One such A is

$$\begin{pmatrix}3 & -6\\-1 & 2\end{pmatrix}$$

but many other examples are possible. For example,

$$\begin{pmatrix}1 & -2\\-1/3 & 2/3\end{pmatrix}$$

c) A^9 just implements A nine times in succession. A rotates vectors cc by $10°$, so A^2 rotates vectors cc by $10(2) = 20°$, and A^3 rotates vectors cc by $10(3) = 30°$, etc.

Thus, A^9 rotates cc by $10(9) = 90°$, so $A^9\begin{pmatrix}2\\0\end{pmatrix}$ is just the cc rotation of $\begin{pmatrix}2\\0\end{pmatrix}$ by $90°$:

$$A^9\begin{pmatrix}2\\0\end{pmatrix} = \begin{pmatrix}0\\2\end{pmatrix}.$$

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.