

Math 1553 Worksheet §5.6 - §6.5

1. Courage Soda and Dexter Soda compete for a market of 210 customers who drink soda each day.

Today, Courage has 80 customers and Dexter has 130 customers. Each day:

70% of Courage Soda's customers keep drinking Courage Soda, while 30% switch to Dexter Soda.

40% of Dexter Soda's customers keep drinking Dexter Soda, while 60% switch to Courage Soda.

- a) Write a stochastic matrix A and a vector x so that Ax will give the number of customers for Courage Soda and Dexter Soda (in that order) tomorrow. You do not need to compute Ax .

- b) By finding the 1-eigenspace, work shows that the steady state vector is

$$w = \begin{pmatrix} 2/3 \\ 1/3 \end{pmatrix}.$$

Using this determine the following: in the long run, roughly how many daily customers will Courage Soda have?

2. a) Find the standard matrix B for proj_W , where $W = \text{Span} \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right\}$.

b) What are the eigenvalues of B ? Is B diagonalizable?

c) Let $x = \begin{pmatrix} 3 \\ 0 \\ 9 \end{pmatrix}$. Find the orthogonal decomposition of x with respect to W .

In other words, find x_W in W and x_{W^\perp} in W^\perp so that $x = x_W + x_{W^\perp}$.

3. Use least-squares to find the best fit line $y = Ax + B$ through the points $(0, 0)$, $(1, 8)$, $(3, 8)$, and $(4, 20)$.