Supplemental problems: §5.5

1. **a)** If A is the matrix that implements rotation by $143°$ in $\mathbf{R}^2$, then A has no real eigenvalues.

b) A 3×3 matrix can have eigenvalues 3, 5, and $2+i$.

c) If $v = \begin{pmatrix} 2+i \\ 1 \end{pmatrix}$ is an eigenvector of A corresponding to the eigenvalue $\lambda = 1-i$, then $w = \begin{pmatrix} 2i-1 \\ i \end{pmatrix}$ is an eigenvector of A corresponding to the eigenvalue $\lambda = 1-i$.

2. Consider the matrix

$$A = \begin{pmatrix} 3\sqrt{3}-1 & -5\sqrt{3} \\ 2\sqrt{3} & -3\sqrt{3}-1 \end{pmatrix}$$

a) Find both complex eigenvalues of A.

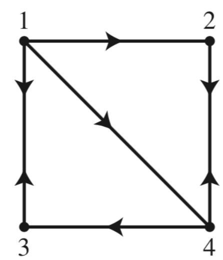
b) Find an eigenvector corresponding to each eigenvalue.

3. This problem is an example of a 3×3 matrix that has a mix of real and (non-real) complex eigenvalues. In such a case, we are not able to use the "2×2 eigenvector trick" because the matrix is 3×3, and so we would need to do row-reduction to find the complex eigenvectors. This particular kind of problem is computationally beyond the scope of what to expect on a Math 1553 exam in Spring 2022.

Let $A = \begin{pmatrix} 4 & -3 & 3 \\ 3 & 4 & -2 \\ 0 & 0 & 2 \end{pmatrix}$. Find all eigenvalues of A. For each eigenvalue of A, find a corresponding eigenvector.

Supplemental problems: §5.6

1. Suppose the internet has four pages in the following manner. Arrows represent links from one page towards another. For example, page 1 links to page 4 but not vice versa.



a) Write the importance matrix and the Google matrix for this internet using damping constant $p = 0.15$. You don't need to simplify the Google matrix.

b) The steady-state vector for the Google matrix is (approximately)

$$\begin{pmatrix} 0.23 \\ 0.23 \\ 0.23 \\ 0.31 \end{pmatrix}.$$

What is the top-ranked page?

2. The companies X, Y, and Z fight for customers. This year, company X has 40 customers, Company Y has 15 customers, and Z has 20 customers. Each year, the following changes occur:

- X keeps 75% of its customers, while losing 15% to Y and 10% to Z.
- Y keeps 60% of its customers, while losing 5% to X and 35% to Z.
- Z keeps 65% of its customers, while losing 15% to X and 20% to Y.

Write a stochastic matrix A and a vector x so that Ax will give the number of customers for firms X, Y, and Z (respectively) after one year. You do not need to compute Ax.

3. Courage Soda and Dexter Soda compete for a market of 210 customers who drink soda each day.
Today, Courage has 80 customers and Dexter has 130 customers. Each day:

70% of Courage Soda's customers keep drinking Courage Soda, while 30% switch to Dexter Soda.

40% of Dexter Soda's customers keep drinking Dexter Soda, while 60% switch to Courage Soda.

a) Write a stochastic matrix A and a vector x so that Ax will give the number of customers for Courage Soda and Dexter Soda (in that order) tomorrow.
You do not need to compute Ax.

b) Find the steady-state vector for A.

c) Use your answer from (b) to determine the following: in the long run, roughly how many daily customers will Courage Soda have?