MATH 1553, EXAM 3 SOLUTIONS
SPRING 2023

Name		**GT ID**	

Circle your lecture below.

Jankowski, lec. A and HP (8:25-9:15 AM) Jankowski, lecture D (9:30-10:20 AM)

Sane, lecture G (12:30-1:20 PM)

Sun, lecture I (2:00-2:50 PM) Sun, lecture M (3:30-4:20 PM)

Please **read all instructions** carefully before beginning.

- Write your initials at the top of each page.
- The maximum score on this exam is 70 points, and you have 75 minutes to complete this exam. Each problem is worth 10 points.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means "reduced row echelon form."
- The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and all work must be written on the exam itself, with no exceptions.
- Unless stated otherwise, **the entries of all matrices on the exam are real numbers**.
- This exam is double-sided. You should have more than enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam*. If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Please read and sign the following statement.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 7:45 PM on Wednesday, April 12.

____

This page was intentionally left blank.

Problem 1.

For each statement, answer TRUE or FALSE. If the statement is *ever* false, circle FALSE. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

a) Suppose S is a rectangle in $\mathbf{R}^2$ with area 2, and let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the matrix transformation

$$T(x) = \begin{pmatrix} 1 & 0 \\ 15 & 3 \end{pmatrix} x.$$

Then the area of $T(S)$ is 6.

$\boxed{\text{TRUE}}$ FALSE

b) Suppose that $\lambda = -5$, $\lambda = 1$, and $\lambda = 8$ are eigenvalues of a 5×5 matrix A. If $\dim(\text{Null}(A - 8I)) = 3$, then A must be diagonalizable.

$\boxed{\text{TRUE}}$ FALSE

c) There is an $n \times n$ matrix A so that the zero vector is an eigenvector of A.

TRUE $\boxed{\text{FALSE}}$

d) Let $A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{pmatrix}$. Then $\lambda = 0$ is an eigenvalue of A.

$\boxed{\text{TRUE}}$ FALSE

e) Let A be the 2×2 matrix that rotates vectors in $\mathbf{R}^2$ by 65 degrees counterclockwise. Then A has no real eigenvalues. $\boxed{\text{TRUE}}$ FALSE

Solution.

a) True: the area is $2|\det(A)| = 2(1(3) - 15(0)) = 6$. This problem is essentially #9 in the Webwork for Determinants I.

b) True: We are told that the 8-eigenspace is 3-dimensional. We are also told that $\lambda = 1$ and $\lambda = -5$ are eigenvalues, so their eigenspaces must be at least 1-dimensional. This guarantees that A has 5 linearly independent eigenvectors in $\mathbf{R}^5$, therefore A is diagonalizable. This is similar to many diagonalization problems we have done, for example #5 in Sample Midterm 3A.

c) False: the zero vector is **never** an eigenvector of any matrix. This is a fundamental emphasis of Math 1553.

d) True: one row-replacement shows A is not invertible because it has only two pivots.

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{pmatrix} \xrightarrow{R_3 = R_3 + R_1} \begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix} \xrightarrow{R_3 = R_3 - R_2} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}.$$

The fact that A is not invertible is precisely the same thing as saying that $Ax = 0x$ has infinitely many solutions, so $\lambda = 0$ is an eigenvalue of A.

e) True: if x is in $\mathbf{R}^2$ and not the zero vector, then x and Ax are on different lines through the origin, so neither is a scalar multiple of the other.

Problem 2.

Parts (a), (b), (c), and (d) are unrelated. On (a) and (b), you do not need to show your work, and there is no partial credit. Show your work on (c) and (d).

a) (2 points) Suppose $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 7$. Find $\det\begin{pmatrix} d & e & f \\ a & b & c \\ -2a+g & -2b+h & -2c+i \end{pmatrix}$.

Clearly circle the correct answer below.

(i) 7 (ii) −7 (iii) 14 (iv) −14

(v) 56 (vi) −56 (vii) not enough information (viii) none of these

b) (2 points) Write a 2×2 matrix A that is diagonalizable but not invertible.

c) (3 points) Find the area of the triangle with vertices $(1,2)$, $(2,3)$, and $(4,-5)$.

d) (3 points) Find all values of a so that $\det\begin{pmatrix} 1 & -3 & 0 \\ 1 & a & 0 \\ a & 0 & a \end{pmatrix} = 0$.

Solution.

a) This problem was nearly copied from #7 in the Webwork for Determinants I. The final matrix is obtained from the original matrix by swapping the first two rows (multiplying the determinant by −1) and then doing the row replacement operation of adding −2(second row) to the third row (does not change the determinant). Therefore, the determinant of the final matrix is $7(-1) = -7$.

b) Many examples possible. For example, we can choose any diagonal 2×2 matrix that has at least one diagonal entry equal to 0.

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad A = \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 5 \\ 0 & 1 \end{pmatrix}.$$

c) This problem is #8 in the Webwork for Determinants I, with changed numbers. We can choose our favorite starting vertex and form vectors v_1 and v_2 from that vertex to the others.

v_1 from $(1,2)$ to $(2,3)$: $v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

v_2 from $(1,2)$ to $(4,-5)$: $v_2 = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$.

$$\text{Area} = \frac{1}{2}\left|\det\begin{pmatrix} 1 & 3 \\ 1 & -7 \end{pmatrix}\right| = \frac{1}{2}|-7-3| = \frac{1}{2}(10) = 5.$$

d) This problem was taken from #1 in the Webwork for Determinants II, with changed numbers. We need $1(a^2-0)-(-3)(a-0)+0 = 0$, so $a^2+3a = a(a+3) = 0$. Therefore, $a = 0$ and $a = -3$ are the values that make the determinant equal to 0.

Problem 3.

Parts (a), (b), and (c) are unrelated. You do not need to show your work on this page, and there is no partial credit.

a) (3 points) Let A and B be 3×3 matrices satisfying $\det(A) = 2$ and $\det(B) = -3$. Which of the following must be true? Clearly circle all that apply.

(i) $\det(A+B) = \det(A) + \det(B)$.

$\boxed{\text{(ii)}}$ $\det(A^T B^{-1}) = -2/3$.

$\boxed{\text{(iii)}}$ $\det(-2A) = -16$.

b) (4 points) Suppose A is an $n \times n$ matrix. Which of the following conditions guarantee that $\lambda = 4$ is an eigenvalue of A? Clearly circle all that apply.

$\boxed{\text{(i)}}$ The equation $(A - 4I)x = 0$ has infinitely many solutions.

(ii) There is a nonzero vector x in $\mathbf{R}^n$ so that the set $\{x, Ax\}$ is linearly dependent.

$\boxed{\text{(iii)}}$ There is a non-trivial solution to the equation $Ax = 4x$.

(iv) $\text{Nul}(A - 4I) = \{0\}$.

c) (3 points) Suppose A is a 3×3 matrix with characteristic polynomial

$$\det(A - \lambda I) = -\lambda(\lambda + 1)^2.$$

Which of the following statements are true? Clearly circle all that apply.

$\boxed{\text{(i)}}$ The eigenvalues of A are -1 and 0.

(ii) A cannot be diagonalizable.

$\boxed{\text{(iii)}}$ The null space of A must be 1-dimensional.

Solution.

a) We took (i) directly from #8 in the Webwork for Determinants II. It is rarely true that $\det(A+B) = \det(A) + \det(B)$, for example if $A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix}$

then $\det(A) + \det(B) = 1$ but $\det(A + B) = -6$.
(ii) was essentially taken from #1 on the 4.1-5.1 Worksheet and almost identical to #2a on Sample Midterm 3A. It is true, since

$$\det(A^T B^{-1}) = \det(A^T)\det(B^{-1}) = \det(A) \cdot \frac{1}{\det(B)} = 2 \cdot \frac{-1}{3} = -2/3.$$

(iii) is an application of #2 from the 4.1-5.1 Worksheet. It is true since

$$\det(-2A) = (-2)^3 \det(A) = -8(2) = -16.$$

b) (i) and (iii) are true by the definition of eigenvalue, but (ii) is not necessarily true (it just means A has a real eigenvalue), and (iv) is the statement that 4 is NOT an eigenvalue.

c) (i) is true since $\lambda = 0$ and $\lambda = -1$ are the values of λ that satisfy the characteristic equation.
(ii) was essentially taken from #1b in the 5.2-5.4 Worksheet. It is not necessarily true because A might be diagonalizable, for example

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

(iii) is true and is a classic fact: the algebraic multiplicity of $\lambda = 0$ is 1, therefore the geometric multiplicity of $\lambda = 0$ (aka the dimension of the 0-eigenspace, aka the dimension of the null space of A) must also be 1.

Problem 4.

Parts (a), (b), and (c) are unrelated. Briefly show your work on (c).

a) Let A be the 3×3 matrix for projection onto the xy-plane in $\mathbf{R}^3$.
(i) (2 points) What are the eigenvalues of A?

The eigenvalues are $\lambda = 0$ and $\lambda = 1$.

(ii) (1 point) Is A diagonalizable? $\boxed{\text{YES}}$ NO

b) (4 points) Let A be the 2×2 matrix that reflects vectors across the line $y = x$. Fill in the blanks below.

One eigenvalue of A is $\lambda_1 = \underline{\quad 1 \quad}$ and an eigenvector for λ_1 is $v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

The other eigenvalue of A is $\lambda_2 = \underline{\quad -1 \quad}$ and an eigenvector for λ_2 is $v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

(others are possible, for example $v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ or $v_2 = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ etc.)

c) (3 points) Suppose A is a 2×2 positive stochastic matrix with the property that as n gets very large, $A^n \begin{pmatrix} 90 \\ 70 \end{pmatrix}$ approaches $\begin{pmatrix} 120 \\ 40 \end{pmatrix}$.

What is the steady-state vector w for A? $w = \begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix}$

Solution.

a) This part was taken from #3a in the 4.1-5.1 Worksheet. (i) The eigenvalues are $\lambda = 0$ and $\lambda = 1$, because $Av = 0$ for all v along the z-axis and $Av = v$ for all v in the xy-plane. These eigenvalues already give 3 linearly independent eigenvectors in $\mathbf{R}^3$, so there cannot be any more eigenvalues.

(ii) Yes. In fact, A is already *diagonal*! $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

b) This was taken from #3b in the 4.1-5.1 Worksheet and one of the sample exams.

One eigenvalue is $\lambda_1 = 1$, and one possibility for v_1 is $v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ since A fixes all vectors along the line $y = x$.

$\lambda_2 = -1$, and one possibility for v_2 is $v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ since A flips all vectors along the line through the origin perpendicular to $y = x$ (this is the line $y = -x$).

c) This is #4a from Sample Midterm 3B with changed numbers. By the Perron-Frobenius Theorem, $A^n \begin{pmatrix} 90 \\ 70 \end{pmatrix}$ approaches 160 times the steady-state vector w, so

$$160w = \begin{pmatrix} 120 \\ 40 \end{pmatrix}, \qquad w = \begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix}.$$

Problem 5.

Free response. Show your work unless otherwise indicated! A correct answer without sufficient work may receive little or no credit.

For this problem, let $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 4 & 4 \\ 0 & 0 & 3 \end{pmatrix}$.

a) (2 points) Find the eigenvalues of A. You do not need to show your work on this part.
This page is a quintessential diagonalization problem. In fact, it is nearly #6 verbatim from both Sample Midterms (3A and 3B).
For part (a), A is upper-triangular, so its eigenvalues are its diagonal entries: $\lambda = 3$ and $\lambda = 4$.

b) (5 points) For each eigenvalue of A, find a basis for the corresponding eigenspace.

$$(A - 3I|0) = \left(\begin{array}{ccc|c} 0 & 1 & 4 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_2 = R_2 - R_1} \left(\begin{array}{ccc|c} 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

This gives x_1 free, $x_2 = -4x_3$, and x_3 free.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ -4x_3 \\ x_3 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}. \quad \boxed{\text{Basis for 3-eigenspace} : \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix} \right\}}.$$

$$(A - 4I|0) = \left(\begin{array}{ccc|c} -1 & 1 & 4 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right) \xrightarrow{\text{3rd column ops}} \left(\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

This gives $x_1 = x_2$, x_2 free, and $x_3 = 0$.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_2 \\ 0 \end{pmatrix} = x_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad \boxed{\text{Basis for 4-eigenspace} : \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}}.$$

c) (3 points) The matrix A is diagonalizable. Write a 3×3 matrix C and a 3×3 diagonal matrix D so that $A = CDC^{-1}$. Enter your answer below.

We form C using linearly independent eigenvectors and form D using the eigenvalues written **in the corresponding order**. Many answers are possible. For example,

$$C = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

or

$$C = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -4 \\ 0 & 0 & 1 \end{pmatrix}, \qquad D = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Problem 6.

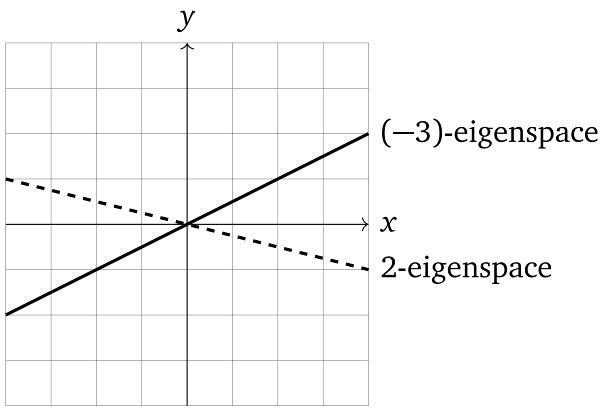
Free response. Fully simplify your answers. Parts (a) and (b) are unrelated. Show your work! A correct answer without sufficient work may receive little or no credit.

a) Let $A = \begin{pmatrix} 1 & -5 \\ 2 & 3 \end{pmatrix}$.

(i) (3 points) Find the complex eigenvalues of A.

(ii) (3 pts) For the eigenvalue with positive imaginary part, find an eigenvector v.

b) (4 points) Let A be the 2×2 matrix whose (-3)-eigenspace is the **solid** line below and whose 2-eigenspace is the **dashed** line below.



Find $A\begin{pmatrix} 6 \\ 0 \end{pmatrix}$. Enter your answer here: $A\begin{pmatrix} 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

Solution.

a) (i) The characteristic polynomial is

$$\det(A - \lambda I) = (\lambda - 1)(\lambda - 3) - (2)(-5) = \lambda^2 - 4\lambda + 13.$$

The eigenvalues are $\lambda = \dfrac{4 \pm \sqrt{(-4)^2 - 4(13)(1)}}{2} = \dfrac{4 \pm \sqrt{-36}}{2} = 2 \pm 3i$.

(ii) For $\lambda = 2 + 3i$, we use the 2×2 trick: If the first row of $A - \lambda I$ is $\begin{pmatrix} a & b \end{pmatrix}$ (where a and b are not both zero), then $\begin{pmatrix} -b \\ a \end{pmatrix}$ is an eigenvector of A.

$$\left(A - (2+3i)I \mid 0\right) = \left(\begin{array}{cc|c} -1-3i & -5 & 0 \\ (*) & (*) & 0 \end{array}\right),$$

so $v = \begin{pmatrix} 5 \\ -1-3i \end{pmatrix}$ is an eigenvector. An equivalent answer is $\begin{pmatrix} -5 \\ 1+3i \end{pmatrix}$ which is just $-1*$(the previous answer for v).

b) This is #5a from Sample Midterm 3A with changed numbers. The graph says that (-3)-eigenspace is Span$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and the 2-eigenspace is Span$\begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

We can solve or just observe $\begin{pmatrix}6\\0\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix} + \begin{pmatrix}4\\-1\end{pmatrix}$, so

$$\begin{aligned} A\begin{pmatrix}6\\0\end{pmatrix} &= A\left(\begin{pmatrix}2\\1\end{pmatrix} + \begin{pmatrix}4\\-1\end{pmatrix}\right) = A\begin{pmatrix}2\\1\end{pmatrix} + A\begin{pmatrix}4\\-1\end{pmatrix} \\ &= -3\begin{pmatrix}2\\1\end{pmatrix} + 2\begin{pmatrix}4\\-1\end{pmatrix} = \begin{pmatrix}-6\\-3\end{pmatrix} + \begin{pmatrix}8\\-2\end{pmatrix} = \begin{pmatrix}2\\-5\end{pmatrix}. \end{aligned}$$

Problem 7.

Free response. Show your work! A correct answer without sufficient work may receive little or no credit. Parts (a) and (b) are unrelated.

a) Awesome Coffee and Bunk Coffee compete for a market of 60 customers who drink coffee every day. Today, Awesome Coffee has 50 daily customers and Bunk Coffee has 10 daily customers.
Each day:

- 60% of Awesome Coffee customers keep drinking Awesome Coffee, while 40% switch to Bunk Coffee.
- 80% of Bunk Coffee customers keep drinking Bunk Coffee, while 20% switch to Awesome Coffee.

(i) (3 points) Write a positive stochastic matrix A **and** a vector x so that Ax will give the number of customers for Awesome Coffee and Bunk Coffee (in that order) tomorrow. You do not need to compute Ax.

$$A = \begin{pmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{pmatrix} \qquad x = \begin{pmatrix} 50 \\ 10 \end{pmatrix}$$

(ii) (4 points) In the long term, approximately how many daily customers will Awesome Coffee have?

To receive full credit, you needed to answer the question that was directly asked. This means that your answer must be a **number**, not a vector. If you just wrote $\begin{pmatrix} 20 \\ 40 \end{pmatrix}$ and did not interpret that vector to answer the question, you did not receive full credit.

$$(A - I|0) = \left(\begin{array}{cc|c} -0.4 & 0.2 & 0 \\ 0.4 & -0.2 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} -0.4 & 0.2 & 0 \\ 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -1/2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

so $x_1 = \frac{1}{2}x_2$ and the 1-eigenspace is Span$\begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$. The steady state vector is therefore

$$w = \frac{1}{1+\frac{1}{2}}\begin{pmatrix} 1/2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix}.$$

This means **Awesome Coffee** will get about 1/3 of the 60 total customers in the long run, so **20 customers** (whereas Bunk Coffee will get about 2/3 of the 60 total customers, so 40 in the long run).

Another way to see this more rigorously is to note that, as n gets large,

$$A^n \begin{pmatrix} 50 \\ 10 \end{pmatrix} \longrightarrow 60\begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix} = \begin{pmatrix} 20 \\ 40 \end{pmatrix},$$

so in the long run Awesome Coffee will have about 20 daily customers.

b) (3 points) Find $\det\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 1 & -1 & 1 \\ 0 & 0 & 0 & 4 \\ 0 & 6 & 0 & 1 \end{pmatrix}$.

Cofactor expansion along the third row gives

$$\begin{aligned} 4(-1)^{3+4}\det\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -1 \\ 0 & 6 & 0 \end{pmatrix} &= -4\det\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -1 \\ 0 & 6 & 0 \end{pmatrix} \\ &= -4\cdot 6(-1)^{3+2}\det\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \\ &= (-4)(-6)(-1+2) = 24. \end{aligned}$$

Alternatively, we could have done the standard 3×3 determinant formula after our initial cofactor expansion.

$$\begin{aligned} 4(-1)^{3+4}\det\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -1 \\ 0 & 6 & 0 \end{pmatrix} &= -4\det\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -1 \\ 0 & 6 & 0 \end{pmatrix} \\ &= -4\Big[1(6) - 2(0) - 1(12)\Big] \\ &= -4(-6) = 24. \end{aligned}$$

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.