Math 1553 Worksheet §2.5, 2.6, 2.7, 2.9, 3.1

1. If the statement is always true, circle TRUE. Otherwise, circle FALSE. Justify your answer.

   a) Suppose $A = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$ and $A\begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$. Must v_1, v_2, v_3 be linearly dependent? If yes, write a linear dependence relation for the vectors.

   YES   **NO**

   b) If b is in Col(A), then so is $5b$.   **TRUE**   **FALSE**

   c) In the following, A is an $m \times n$ matrix.

   (1) **TRUE**   **FALSE**   If A has linearly dependent columns, then $m < n$.

   (2) **TRUE**   **FALSE**   If A has linearly independent columns, then $Ax = b$ must have at least one solution for each b in $\mathbf{R}^m$.

   (3) **TRUE**   **FALSE**   If b is a vector in $\mathbf{R}^m$ and $Ax = b$ has exactly one solution, then $m \geq n$.

2. Circle **TRUE** if the statement is always true, and circle **FALSE** otherwise.

    a) If A is a 3×10 matrix with 2 pivots, then $\dim(\text{Nul}\,A) = 8$ and $\text{rank}(A) = 2$.

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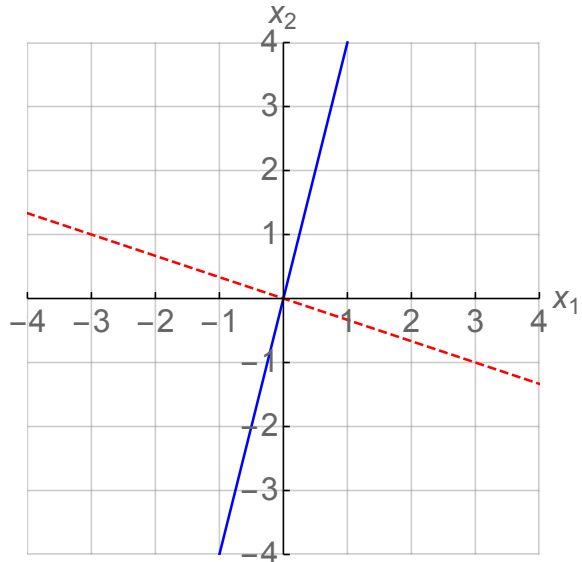**TRUE**   **FALSE**

</div>

    b) If A is an $m \times n$ matrix and $Ax = 0$ has only the trivial solution, then the transformation $T(x) = Ax$ must have $\mathbf{R}^m$ as its range.

<div align="center">

TRUE   **FALSE**

</div>

    c) If $\{a, b, c\}$ is a basis of a subspace V, then $\{a, a + b, b + c\}$ is a basis of V as well.

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TRUE   **FALSE**

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3. Write a matrix A so that $\text{Col}(A)$ is the solid blue line and $\text{Nul}(A)$ is the dotted red line drawn below.

4. Let $A = \begin{pmatrix} 1 & -5 & -2 & -4 \\ 2 & 3 & 9 & 5 \\ 1 & 1 & 4 & 2 \end{pmatrix}$, and let T be the matrix transformation associated to

A, so $T(x) = Ax$.

a) What is the domain of T? What is the codomain of T? Give an example of a vector in the range of T.

b) The RREF of A is $\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

(i) Write bases for Col(A) and Nul(A).

(ii) Is there a vector in the codomain of T which is not in the range of T? Justify your answer.