Math 1553 Worksheet, Chapter 3

1. Let $A = \begin{pmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{pmatrix}$.

a) Compute $\det(A)$ using row reduction.

b) Compute $\det(A^{-1})$ without doing any more work.

c) Compute $\det((A^T)^5)$ without doing any more work.

2. Compute the determinant of

$$A = \begin{pmatrix} 4 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 3 & 0 & 0 & 0 \\ 8 & 3 & 1 & 7 \end{pmatrix}$$

using cofactor expansions. Expand along the rows or columns that require the least amount of work.

3. If A is a 3×3 matrix and $\det(A) = 1$, what is $\det(2A)$?

Supplemental Problems

For those who want additional practice problems after completing the worksheet, here are some extra practice problems.

1. Let A be an $n \times n$ matrix.

a) Using cofactor expansion, explain why $\det(A) = 0$ if A has a row or a column of zeros.

b) Using cofactor expansion, explain why $\det(A) = 0$ if A has adjacent identical columns.

2. Find the volume of the parallelepiped naturally formed by $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$.

3. Is there a 3×3 matrix A with only real entries, such that $A^4 = -I$? Either write such an A, or show that no such A exists.

4. Find the inverse of

$$A = \begin{pmatrix} 4 & 1 & 4 \\ 3 & 0 & 2 \\ 0 & 5 & 0 \end{pmatrix}$$

using the formula

$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}$$