MATH 1553, C.J. JANKOWSKI
MIDTERM 1

Name		Section	

Please **read all instructions** carefully before beginning.

- You have 50 minutes to complete this exam.
- There are no aids of any kind (calculators, notes, text, etc.) allowed.
- Please show your work unless specified otherwise. A correct answer without appropriate work may be given little or no credit.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Scoring Page

Please do not write on this page.

1	2	3	4	5	Total

Problem 1.

[Parts a) through f) are worth 2 points each]

a) Compute $\begin{pmatrix} 3 & 2 \\ -2 & 0 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

b) If A is a 2×3 matrix with 2 pivots, then the set of solutions to $Ax = 0$ is a:

(circle one answer) point line 2-plane 3-plane

in:

(circle one answer) $\mathbf{R}$ $\mathbf{R}^2$ $\mathbf{R}^3$.

True or false. Circle **T** if the statement is **always** true, and circle **F** otherwise. You do not need to justify your answer.

c) **T** **F** If a system of linear equations has more variables than equations, then the system must have infinitely many solutions.

d) **T** **F** If A is an $m \times n$ matrix and A has a pivot in every column, then the equation $Ax = b$ has a solution for each b in $\mathbf{R}^m$.

e) **T** **F** The three vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ span $\mathbf{R}^3$.

f) **T** **F** If u, v, and w are nonzero vectors in $\mathbf{R}^2$, then Span$\{u, v, w\}$ is $\mathbf{R}^2$.

Problem 2.

[10 points]

Johnny Rico believes that the secret to the universe can be found in the system of two linear equations in x and y given by

$$x - y = h$$
$$3x + hy = 4$$

where h is a real number.

a) Find all values of h (if any) which make the system inconsistent. Briefly justify your answer.

b) Find all values of h (if any) which make the system have a unique solution. Briefly justify your answer.

Problem 3. [11 points]

a) Solve the system of equations by putting an augmented matrix into reduced row echelon form. Clearly indicate which variables (if any) are free variables.

$$x_1 + 2x_2 + 2x_3 - x_4 = 4$$
$$2x_1 + 4x_2 + x_3 - 2x_4 = -1$$
$$-x_1 - 2x_2 - x_3 + x_4 = -1$$

b) Write the set of solutions to

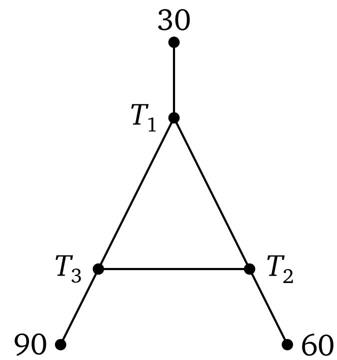
$$x_1 + 2x_2 + 2x_3 - x_4 = 0$$
$$2x_1 + 4x_2 + x_3 - 2x_4 = 0$$
$$-x_1 - 2x_2 - x_3 + x_4 = 0$$

in parametric vector form.

Problem 4.

[10 points]

The diagram below represents the temperature at points along wires, in celcius.



Let T_1, T_2, T_3 be the temperatures at the interior points. Assume the temperature at each interior point is the average of the temperatures of the three adjacent points.

a) Write a system of three linear equations whose solution would give the temperatures T_1, T_2, and T_3. Do not solve it.

b) Write the system as a vector equation. Do not solve it.

c) Write a matrix equation $Ax = b$ that represents this system. Specify every entry of A, x, and b. Do not solve it.

Problem 5.

[6 points]

Write an augmented matrix corresponding to a system of two linear equations in three variables x_1, x_2, x_3, whose solution set is the span of $\begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}$.

Briefly justify your answer.

[Scratch work]