MATH 1553
SAMPLE MIDTERM 1: THROUGH 1.5

Name		**Section**	

1	2	3	4	5	Total

Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. I recommend completing the practice exam in 50 minutes, without notes or distractions.

Problem 1.

[2 points each]

In this problem, A is an $m \times n$ matrix (m rows and n columns) and b is a vector in $\mathbf{R}^m$. Circle **T** if the statement is always true (for any choices of A and b) and circle **F** otherwise. Do not assume anything else about A or b except what is stated.

a) **T** **F** The matrix below is in reduced row echelon form.

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & -3 & 1 \\ 0 & 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

b) **T** **F** If A has fewer than n pivots, then $Ax = b$ has infinitely many solutions.

c) **T** **F** If the columns of A span $\mathbf{R}^m$, then $Ax = b$ must be consistent.

d) **T** **F** If $Ax = b$ is consistent, then the equation $Ax = 5b$ is consistent.

e) **T** **F** If $Ax = b$ is consistent, then the solution set is a span.

Problem 2.

[5 points each]

Acme Widgets, Gizmos, and Doodads has two factories. Factory A makes 10 widgets, 3 gizmos, and 2 doodads every hour, and factory B makes 4 widgets, 1 gizmo, and 1 doodad every hour.

a) If factory A runs for a hours and factory B runs for b hours, how many widgets, gizmos, and doodads are produced? Express your answer as a vector equation.

b) A customer places an order for 16 widgets, 5 gizmos, and 3 doodads. Can Acme fill the order with no widgets, gizmos, or doodads left over? If so, how many hours do the factories run? If not, why not?

Problem 3. [10 points]

Consider the system below, where h and k are real numbers.

$$x + 3y = 2$$
$$3x - hy = k.$$

a) Find the values of h and k which make the system inconsistent.

b) Find the values of h and k which give the system a unique solution.

c) Find the values of h and k which give the system infinitely many solutions.

Problem 4.

[10 points]

Consider the following consistent system of linear equations.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= -2 \\ 3x_1 + 4x_2 + 5x_3 + 6x_4 &= -2 \\ 5x_1 + 6x_2 + 7x_3 + 8x_4 &= -2 \end{aligned}$$

a) [4 points] Find the parametric vector form for the general solution.

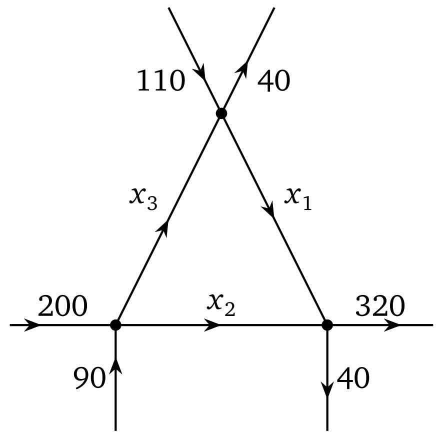
b) [3 points] Find the parametric vector form of the corresponding *homogeneous* equations.

c) [3 points] Unrelated to parts (a) and (b).
If b, v, w are vectors in $\mathbf{R}^3$ and $\text{Span}\{b, v, w\} = \mathbf{R}^3$, is it possible that b is in $\text{Span}\{v, w\}$? Fully justify your answer.

Problem 5. [10 points]

The diagram below describes traffic in a part of town.

Traffic flow (cars/hr)



a) Write a system of three linear equations in x_1, x_2, and x_3 corresponding to the traffic flow.

b) Use an augmented matrix to solve this system of linear equations. Were we given enough information to know the exact values of x_1, x_2, and x_3?

[Scratch work]