MATH 1553
PRACTICE MIDTERM 3 (VERSION A)

Name		**Section**	

1	2	3	4	5	Total

Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is similar in format, length, and difficulty to the real exam. It is not meant as a comprehensive list of study problems. I recommend completing the practice exam in 50 minutes, without notes or distractions.

The exam is not designed to test material from the previous midterm on its own. However, knowledge of the material prior to chapter 3 is necessary for everything we do for the rest of the semester, so it is fair game for the exam as it applies to chapters 3 and 5.

Problem 1.

[2 points each]

In this problem, if the statement is always true, circle **T**; if it is always false, circle **F**; if it is sometimes true and sometimes false, circle **M**.

a) **T** **F** **M** If A is a 3×3 matrix with characteristic polynomial $-\lambda^3 + \lambda^2 + \lambda$, then A is invertible.

b) **T** **F** **M** A 3×3 matrix with (only) two distinct eigenvalues is diagonalizable.

c) **T** **F** **M** If A is diagonalizable and B is similar to A, then B is diagonalizable.

d) **T** **F** **M** A diagonalizable $n \times n$ matrix admits n linearly independent eigenvectors.

e) **T** **F** **M** If $\det(A) = 0$, then 0 is an eigenvalue of A.

Problem 2.

[2 points each]

Give an example of a 2×2 real matrix A with each of the following properties. You need not explain your answer.

a) A has no real eigenvalues.

b) A has eigenvalues 1 and 2.

c) A is invertible but not diagonalizable.

d) A is diagonalizable but not invertible.

e) A is a rotation matrix with real eigenvalues.

Problem 3.

Consider the matrix

$$A = \begin{pmatrix} 4 & 2 & -4 \\ 0 & 2 & 0 \\ 2 & 2 & -2 \end{pmatrix}.$$

a) [4 points] Find the eigenvalues of A, and compute their algebraic multiplicities.

b) [4 points] For each eigenvalue of A, find a basis for the corresponding eigenspace.

c) [2 points] Is A diagonalizable? If so, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. If not, why not?

Problem 4.

Consider the matrix

$$A = \begin{pmatrix} 3 & -5 \\ 2 & -3 \end{pmatrix}.$$

a) [3 points] Find the (complex) eigenvalues of A.

b) [2 points] For each eigenvalue of A, find a corresponding eigenvector.

c) [3 points] Find a rotation-scaling matrix C that is similar to A.

d) [1 point] By what factor does C scale?

e) [1 point] By what angle does C rotate?

Problem 5. [10 points]

Compute the determinant of the matrix

$$\begin{pmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{pmatrix}^3 .$$

[Scratch work]