MATH 1553
PRACTICE MIDTERM 3 (VERSION B)

Name		**Section**	

1	2	3	4	5	Total

Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is similar in format, length, and difficulty to the real exam. It is not meant as a comprehensive list of study problems. I recommend completing the practice exam in 50 minutes, without notes or distractions.

The exam is not designed to test material from the previous midterm on its own. However, knowledge of the material prior to chapter 3 is necessary for everything we do for the rest of the semester, so it is fair game for the exam as it applies to chapters 3 and 5.

Problem 1. [2 points each]

In this problem, if the statement is always true, circle **T**; otherwise, circle **F**.

a) **T** **F** If A is row equivalent to B, then A and B have the same eigenvalues.

b) **T** **F** If A is similar to B, then A and B have the same characteristic polynomial.

c) **T** **F** If A is similar to B, then A and B have the same eigenvectors.

d) **T** **F** If A is diagonalizable, then A has n distinct eigenvalues.

e) **T** **F** Row operations do not change the determinant of a matrix.

Problem 2.

In this problem, you need not explain your answers; just circle the correct one(s).

Let A be an $n \times n$ matrix.

a) [3 points] Which **one** of the following statements is correct?

1. An eigenvector of A is a vector v such that $Av = \lambda v$ for a nonzero scalar λ.
2. An eigenvector of A is a nonzero vector v such that $Av = \lambda v$ for a scalar λ.
3. An eigenvector of A is a nonzero scalar λ such that $Av = \lambda v$ for some vector v.
4. An eigenvector of A is a nonzero vector v such that $Av = \lambda v$ for a nonzero scalar λ.

b) [3 points] Which **one** of the following statements is **not** correct?

1. An eigenvalue of A is a scalar λ such that $A - \lambda I$ is not invertible.
2. An eigenvalue of A is a scalar λ such that $(A - \lambda I)v = 0$ has a solution.
3. An eigenvalue of A is a scalar λ such that $Av = \lambda v$ for a nonzero vector v.
4. An eigenvalue of A is a scalar λ such that $\det(A - \lambda I) = 0$.

c) [4 points] Which of the following 3×3 matrices are necessarily diagonalizable over the real numbers? (Circle all that apply.)

1. A matrix with three distinct real eigenvalues.
2. A matrix with one real eigenvalue.
3. A matrix with a real eigenvalue λ of algebraic multiplicity 2, such that the λ-eigenspace has dimension 2.
4. A matrix with a real eigenvalue λ such that the λ-eigenspace has dimension 2.

Problem 3.

Consider the matrix

$$A = \begin{pmatrix} -1 & -4 & 0 \\ 1 & 3 & 0 \\ 7 & 10 & 2 \end{pmatrix}.$$

a) [4 points] Find the eigenvalues of A, and compute their algebraic multiplicities.

b) [4 points] For each eigenvalue of A, find a basis for the corresponding eigenspace.

c) [2 points] Is A diagonalizable? If so, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. If not, why not?

Problem 4.

Consider the matrix

$$A = \begin{pmatrix} 3\sqrt{3}-1 & -5\sqrt{3} \\ 2\sqrt{3} & -3\sqrt{3}-1 \end{pmatrix}$$

a) [2 points] Find both complex eigenvalues of A.

b) [2 points] Find an eigenvector corresponding to each eigenvalue.

c) [3 points] Find an invertible matrix P and a rotation-scale matrix C such that $A = PCP^{-1}$.

d) [1 point] By what factor does C scale?

e) [2 points] By what angle does C rotate?

Problem 5.

Let

$$A = \begin{pmatrix} 7 & 1 & 4 & 1 \\ -1 & 0 & 0 & 6 \\ 9 & 0 & 2 & 3 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 & 5 & 4 \\ 1 & -1 & -3 & 0 \\ -1 & 0 & 5 & 4 \\ 3 & -3 & -2 & 5 \end{pmatrix}$$

a) [3 points] Compute $\det(A)$.

b) [3 points] Compute $\det(B)$.

c) [2 points] Compute $\det(AB)$.

d) [2 points] Compute $\det(A^2 B^{-1} A B^2)$.

[Scratch work]