Math 1553 Quiz 2: 1.1 and 1.2 (10 points, 10 minutes)

Solutions

1. **a)** (2 points) True or false: If a system of 4 linear equations in 3 variables is consistent, it must have a unique solution. Justify your answer.

b) (3 pts) Write an augmented matrix in reduced row echelon form corresponding to a consistent system of three linear equations in the variables x_1, x_2, x_3, in which x_2 is the only free variable.

Solution.

a) False. Such a consistent system can have fewer than 3 pivot columns, and thus a free variable. For example, the following augmented matrix represents a system of 4 equations in 3 variables which has infinitely many solutions.

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ -1 & -1 & -1 & 0 \end{array}\right) \xrightarrow{\text{row operations}} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

b) The first and third columns below (corr. to x_1 and x_3) are the pivot columns. The system is consistent and x_2 is free.

$$\left(\begin{array}{ccc|c} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

2. (5 points) Solve the following system of equations by putting an augmented matrix in reduced row echelon form. Show your work! If the system has any free variables, clearly indicate them when you write the form of the general solution.

$$2y + 2z = 8$$
$$x - 2z = 1$$
$$-2x + y + 5z = 2$$

Solution.

$$\left(\begin{array}{ccc|c} 0 & 2 & 2 & 8 \\ 1 & 0 & -2 & 1 \\ -2 & 1 & 5 & 2 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} \boxed{1} & 0 & -2 & 1 \\ 0 & 2 & 2 & 8 \\ -2 & 1 & 5 & 2 \end{array}\right) \xrightarrow{R_3 = R_3 + 2R_1} \left(\begin{array}{ccc|c} \boxed{1} & 0 & -2 & 1 \\ 0 & 2 & 2 & 8 \\ 0 & 1 & 1 & 4 \end{array}\right) \xrightarrow{R_2 = \frac{R_2}{2}}$$

$$\left(\begin{array}{ccc|c} \boxed{1} & 0 & -2 & 1 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 1 & 1 & 4 \end{array}\right) \xrightarrow{R_3 = R_3 - R_2} \left(\begin{array}{ccc|c} \boxed{1} & 0 & -2 & 1 \\ 0 & \boxed{1} & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

The RREF shows the system is consistent and z is a free variable (z real).

$$\boxed{x = 1 + 2z \qquad y = 4 - z \qquad z \text{ is free } (z = z)}.$$