Name:________________________ Recitation Section:__________

Math 1553 Quiz 3: 1.3 (10 points, 10 minutes)

Solutions

Show your work and justify answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit.

1. (6 points) Find all values of h (if there are any) so that $\begin{pmatrix} 1 \\ -2 \\ h \end{pmatrix}$ can be written as a linear combination of $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$.

Solution.

The vector equation $x_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ h \end{pmatrix}$ corresponds to the following augmented system, which we row-reduce to solve for h.

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 2 & -1 & 4 & -2 \\ 3 & -1 & 5 & h \end{array}\right) \xrightarrow[R_3=R_3-3R_1]{R_2=R_2-2R_1} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -1 & 2 & -4 \\ 0 & -1 & 2 & h-3 \end{array}\right) \xrightarrow{R_3=R_3-R_2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -1 & 2 & -4 \\ 0 & 0 & 0 & h+1 \end{array}\right).$$

The system is consistent precisely when the rightmost column is *not* a pivot column. The rightmost column fails to be a pivot column when $h+1=0$, so $\boxed{h=-1}$.

(for example, $1\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + 4\begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} + 0\begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$)

2. (4 points) Give an example of nonzero vectors u, v, and w in $\mathbf{R}^2$ so that u is in Span$\{v, w\}$ but v is not in Span$\{u, w\}$. Briefly justify your answer.
(Hint: think geometrically! You can do this problem without messy algebra)

Solution.

Many answers are possible. Just choose u and w which determine the same line through the origin, and choose v be a vector that doesn't lie along that line.

For example, we could choose $u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $w = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$.

We see that $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is in Span$\left\{\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}\right\}$ (which is all of $\mathbf{R}^2$)

but $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is not in Span$\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}\right\}$ (which is just all vectors $\begin{pmatrix} t \\ 0 \end{pmatrix}$ with t real).