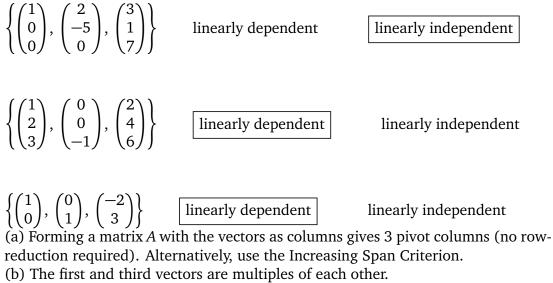
Name:_

Recitation Section:

Math 1553 Quiz 4: 1.7, 1.8, 1.9 (10 points, 10 minutes) Solutions

1. (3 points) In each case, determine whether the set of vectors is linearly dependent or linearly independent. You do not need to justify your answer.



- (c) Three vectors in \mathbf{R}^2 , so automatically linearly dependent.
- 2. Suppose *A* is a 4 × 3 matrix, with corresponding linear transformation T(x) = Ax.
 a) Fill in the blank: The domain of *T* is <u>**R**³</u>.
 - **b)** True or false (no justification required): It is possible that *T* is onto.

TRUE FALSE

It is impossible for any linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ to be onto. It would require *A* to have a pivot in every row, which is impossible since a 4×3 matrix has at most 3 pivots.

3. (5 pts) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation which reflects about the line y = x, then rotates counterclockwise by 45°. Find the matrix *A* so that T(x) = Ax for all *x* in \mathbb{R}^2 . Show your work! Write out the numerical values of any trig functions.

Solution.

It's not necessary to use it in this problem, but the matrix for cc rotation 45° is

$$\begin{pmatrix} \cos(45^\circ) & -\sin(45^\circ) \\ \sin(45^\circ) & \cos(45^\circ) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

We examine what happens to e_1 and e_2 .

$$e_1$$
: The reflection sends $\begin{pmatrix} 1\\0 \end{pmatrix}$ to $\begin{pmatrix} 0\\1 \end{pmatrix}$, then rotating cc by 45° gives $T(e_1) = \begin{pmatrix} \frac{-1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} \end{pmatrix}$.

 e_2 : The reflection sends $\begin{pmatrix} 0\\1 \end{pmatrix}$ to $\begin{pmatrix} 1\\0 \end{pmatrix}$, then rotating cc by 45° gives $T(e_2) = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$.

Therefore,

$$A = \begin{pmatrix} T(e_1) & T(e_2) \end{pmatrix} = \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Alternatively, we could write the reflection matrix $J = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and the rotation

matrix
$$K = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$
, then take the product KJ :

$$KJ = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$