Name:____________________________________ Recitation Section:____________

Math 1553 Quiz 5: 2.1, 2.2, 2.3 (10 points, 10 minutes)

Solutions

Show your work and justify answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit.

1. (3 pts) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be rotation *clockwise* by $90°$. Let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation with standard matrix $\begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}$. Find the standard matrix for the composition $U \circ T$.

Solution.

The matrix for T is $\begin{pmatrix} \cos(-90°) & -\sin(-90°) \\ \sin(-90°) & \cos(-90°) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. The mtx for $U \circ T$ is

$$\begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ 0 & 1 \end{pmatrix}.$$

2. (4 pts) Find A^{-1} if $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{pmatrix}$.

Solution.

We augment $\begin{pmatrix} A & | & I \end{pmatrix}$ and row-reduce.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 3 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3=R_3-2R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right) \xrightarrow{R_1=R_1-R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right).$$

So $A^{-1} = \begin{pmatrix} 3 & 0 & -1 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$.

3. (3 pts) Answer true if the statement is *always* true. Otherwise, answer false.

a) If A and B are matrices and the products AB and BA are both defined, then A and B must be square matrices with the same number of rows and columns.

TRUE **[FALSE]** (example: A is 2×3 and B is 3×2)

b) If A and B are $n \times n$ matrices and $AB = I_n$, then A is invertible and $B = A^{-1}$.

[TRUE] FALSE (taken almost word-for-word from 2.2 class notes).

c) If A is an $n \times n$ matrix and the columns of A are linearly independent, then A is invertible. **[TRUE]** FALSE (A has n pivots)