Midterm 2, Math 2406, Spring 2009

April 8, 2009

1. Suppose that A is a 2×2 matrix with non-negative real entries. Show that A commutes with its transpose (i.e. A^t) under multiplication if and only if $A = A^t$.

2. Compute the determinant of the following matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 9 \\ 3 & 6 & 10 & 12 \\ 4 & 9 & 12 & 16 \end{bmatrix}.$$

3. Suppose that $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation and $v \in \mathbb{R}^n$ is some vector.

a. Fix $v \in \mathbb{R}^n$. Show that $f : \mathbb{R}^n \to \mathbb{R}^n$ given by

$$f(x) \ = \ T(x) + v$$

is a linear transformation if and only if $v = 0$.

b. Mappings of the type f just considered are called "affine maps". Let $A(\mathbb{R}^n, \mathbb{R}^n)$ denote the space of all affine maps, where if f, g are two such maps, with associated transformations T_1 and T_2 and vectors v_1 and v_2, then the mapping $f + g$ is defined to be

$$(f+g)(x) \ = \ (T_1 + T_2)(x) + (v_1 + v_2) \ = \ T_1(x) + T_2(x) + v_1 + v_2.$$

And scalar multiplication is defined similarly. Determine

$$\dim(A(\mathbb{R}^n, \mathbb{R}^n)).$$

4. Use Gaussian elimination to solve the system

$$\begin{aligned} x + 2y + 3z + 4w &= 1 \\ 2x + 5y + 6z + 10w &= 3 \\ 3x + 7y + 9z + 15w &= 5. \end{aligned}$$

5. Define the following terms.

a. linear transformation $f : \mathbb{R}^n \to \mathbb{R}^m$.
b. nullity.
c. cofactor of an $n \times n$ matrix A.
d. surjective map.
e. nullspace.