Linear Algebra, Midterm 2

April 5, 2009

1. Suppose that V is a subspace of $\mathbb{R}^4$ generated by the three vectors $(1, 1, -1, 1), (4, -1, -2, 3)$ and $(4, -7, 0, -1)$. Using the Gram-Schmidt process find three orthogonal vectors that span V.

2. Find the determinant of the following matrix:

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 6 & 1 & 6 \\ 3 & 6 & 6 & 1 \\ 4 & 8 & 7 & 3 \end{bmatrix}.$$

3. There does not exist a vector $x = [x_1\ x_2]^t$ such that

$$\begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}.$$

However, one can find a "closest" vector x (or "least squares" vector) by projecting the vector $[1\ 2\ 2]^t$ onto the column space of the 3×2 matrix on the left-hand-side. Find this projection, and find the corresponding x_1 and x_2 which gives the "closest" vector.

4. Find the eigenvalues and eigenvectors of the following matrix:

$$A = \begin{bmatrix} -7 & 2 \\ -15 & 4 \end{bmatrix}.$$

Then, find a matrix S and a diagonal matrix Λ, such that

$$A = S\Lambda S^{-1}.$$

5. Give an example of a 2×2 matrix A such that

a. $A^8 = I$, but $A^1, A^2, ..., A^7 \neq I$.

b. The matrix A is singular, while the sum of its eigenvalues $\lambda_1 + \lambda_2 = 4$.