

Homework 2, Math 3012, Summer 2010

June 11, 2010

1. Determine the number of integers of the form

$$2^{a_1}3^{a_2}5^{a_3}7^{a_4}11^{a_5}13^{a_6}17^{a_7}19^{a_8},$$

where each a_i is an integer satisfying

$$0 \leq a_1 \leq a_2 \leq \cdots \leq a_8; \text{ and for } i = 1, \dots, 8, a_i \leq i.$$

2. An urn contains 10 black balls and 10 white balls. You randomly select a ball from the urn. If it is black, you stop. If it is white, you keep that white ball, and then select another from the 19 that remain. If *that* ball you just drew is black, you stop. If it is white, you continue drawing balls until a black one is found. What is the probability that you stop after the third draw? (i.e. the third draw is a black ball).

3. Suppose you have a collection of days. 10 percent are rainy days, and 90 percent are dry days. If you select one of the rainy days at random, there is a 20 percent chance it will be a spring day; and if you select a dry day at random, there is an 80 percent chance it won't be a spring day. If you select a spring day at random, what is the probability it was a rainy day?

4. Suppose that $A, B, C,$ and D are independent events (as we defined in class). Prove that

$$\mathbb{P}(A \cap (B \cup C \cup D)) = \mathbb{P}(A)\mathbb{P}(B \cup C \cup D).$$

5. A bag contains 10 red marbles, 20 blue marbles, and 30 white marbles. You select 5 marbles at random WITH REPLACEMENT, which means: you select the first marble, put it back in the bag; then select the second, put it back in the bag; and so on.

What is the probability that 2 marbles are red, 2 are blue, and 1 is white?