

Midterm 1, Math 3012, Summer 2010

June 22, 2010

1. Define the following terms.
 - a. The cross product of sets A and B , denoted by $A \times B$.
 - b. The three axioms of a probability measure. (Kolmogorov Axioms).
 - c. Composition of an integer n .
 - d. Catalan number.
 - e. Multinomial theorem.
2. Determine the number of strings of length 6, using the symbols A through Z, such that the symbols read left-to-right are in alphabetical order. Some examples: BBCEFG, AABBCC, MNXYZZ.
3. You roll a fair die three times. The E be the event that the sum of the numbers from all three rolls is even, and let A be the event that the sum of all three rolls is 10. Find $P(A|E)$. Are events A and E independent?
4. Prove that if A, B and C are independent events, then

$$P(A \cup B \cup C) = 1 - (1 - P(A))(1 - P(B))(1 - P(C)).$$

You may use some major theorems we discussed in class, but you must state them completely and correctly.

5. Prove that for an integer $n \geq 1$ we have

$$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots + \binom{n}{n-\delta} = 2^{n-1}.$$

where

$$\delta = \begin{cases} 0, & \text{if } n \text{ even;} \\ 1, & \text{if } n \text{ odd.} \end{cases}$$