Midterm 2, Math 3215, Summer 2009

July 14, 2009

Instructions: You must supply your own paper. You will be allowed only a simple calculator – no programmable calculators. You have **one hour** to complete the exam.

1. Define the following terms.

a. Say what it means for $X_1, ..., X_k$ to be independent random variables.

b. Define the conditional expectation $\mathbb{E}(X|Y = y)$ in terms of the conditional probability density function $f(x|y)$, and then define $f(x|y)$.

c. Define the moment generating function of a random variable X.

d. Define the mth moment of a random variable X.

e. Define the marginal probability density function for a random variable X, in terms of the joint probability density function for the pair (X, Y).

2.

a. Compute the moment generating function $M_X(t)$ for the random variable X having the probability density function

$$f(x) = \begin{cases} 2x, & \text{if } x \in [0,1]; \\ 0, & \text{otherwise.} \end{cases}$$

b. Using your answer from part a, compute the 3rd moment of X. Don't just write down the answer – explain how you used moment generating functions to find it. (Note: You can easily check your answer, because the 3rd moment is easy to compute directly. This is not always the case, however, as mgf's often provide a much easier way to find moments, than direct computation.)

3. Suppose that (X, Y) is a 2D random variable with probability density function given by $f(x, y) = cx^2y$ when (x, y) is confined to boundary and interior of the triangle with vertices $(0,0),(1,0),(0,1)$; and suppose $f(x,y) = 0$ outside that triangle. Determine the constant c.

4. Suppose you roll a 4-sided fair die (called a "D4"), and then flip two fair coins. Let X be the value of the roll (the numbers 1,2,3,4 are printed on each side of the D4 – the value of your roll is the number printed on the bottom side), and let Y be the number of heads that you flipped.

a. Determine the probability density function of the random variable $Z = X + Y$.

b. Determine the joint probability density function for (X, Z). (One way to do this is to make a 4×3 table of probabilities.)

c. Determine the conditional expectation $\mathbb{E}(X|Z = 4)$.

5. Prove that if X and Y are independent random variables, then

$$V(X + Y) \;=\; V(X) + V(Y).$$