

Makeup midterm exam for Math 3215, Summer 2009

July 22, 2009

Instructions: You will be allowed a simple calculator – no programmable ones, however. You have **one hour** to complete the exam.

1. Define the following terms.
 - a. Percentile of a distribution.
 - b. Gamma function (write down the formula for it).
 - c. The covariance of two random variables.
 - d. Define a χ^2 distribution with n degrees of freedom, in terms of independent normal random variables (like we did in class).
 - e. Say what it means for a collection of random variables X_1, \dots, X_n to be “dependent”.

2.
 - a. Compute the moment generating function $M_X(t)$ for the continuous random variable X having probability density function given by

$$f(x) = \begin{cases} |x|, & \text{if } -1 \leq x \leq 1; \\ 0, & \text{otherwise.} \end{cases}$$

- b. Using your answer from part a, compute the third moment of X . (Note: You can easily check your answer by just computing $\mathbb{E}(X^3)$ directly.)

3. Find the constant c which makes the following function $f(x, y)$ into a probability density function.

$$f(x, y) = \begin{cases} x^2 + cxy, & \text{if } 0 \leq x \leq 1, \text{ and } 0 \leq y \leq 2; \\ 0, & \text{otherwise.} \end{cases}$$

4. Suppose that X has a Poisson distribution with parameter $\lambda = 1$, and let Y be the number of tails that result upon flipping two fair coins. Suppose that X and Y are independent.

a. Determine the probability density function (write it down in a compact form) for the random variable $Z = XY$.

b. Determine the conditional expectation $\mathbb{E}(X|Z = 0)$.

5. Prove that if X and Y are random variables, then

$$V(X + Y) = V(X) + V(Y) + 2\text{Cov}(X, Y).$$