

Midterm 1, Math 3225, Fall 2005

September 3, 2008

1. (10 points)
 - a. List Kolmogorov's axioms of probability.
 - b. Define what it means for Σ to be a sigma-algebra.
 - c. Define what it means for a function φ to be injective, surjective, and bijective.
 - d. Define the product rule of probability.

2. (30 points) An army interrogator, knowing that lie detector tests are only 80% reliable (meaning that if a liar is given the test, the test responds "liar" 80% of the time; and if a truth teller is given the test, the test responds "truth teller" 80% of the time), believes that testing small populations is key to upping the accuracy of the test. He further makes the following bold claim: "Suppose I take 10 suspected terrorists, and ask them if they are part of a terror cell. Suppose they all say "No", and suppose that the lie detector says that 8 of them are lying and 2 are telling the truth. Then, I can be almost 100% certain that at least one of them is a terrorist. Thus, I would hold all ten and question them further."

Let's check the army 'expert': Suppose you know that 3% of suspected terrorists are actually terrorists, and suppose that every suspected terrorist given the test always says "No, I am not a terrorist."

- a. First, use Bayes's Theorem to calculate the probability that a particular suspect is a terrorist given that the test comes back "liar".
- b. Next, calculate the probability that a particular suspect is a terrorist given that the test comes back "truth teller".
- c. Then, calculate the probability p that none of the 10 suspects are terrorists, given that 8 of them are deemed "liar"s by the test, and 2 are deemed "truth teller"s by the test.

3. (20 points) The number of people arriving at a fast food drive through in any given 2 minute interval obeys a Poisson process with mean 1. Suppose that the waiters can only process 3 orders in any given 4 minute interval. What is the expected number of people that leave the drive through with their orders filled in any given 4 minute time interval. Recall that X is a Poisson random variable with mean λ means that $P(X = j) = e^{-\lambda}\lambda^j/j!$.
4. (20 points) Let $P(t)$ denote the price of a stock at time t . Suppose that the price fluctuates according to the following rule: There is a 50% chance that the stock price will double from one week to the next, and a 50% chance it will drop by half from one week to the next. What is the probability that after 6 weeks the price of the stock is 4 times what it was at the start of the 6 week interval.
5. (20 points) Suppose that (X, Y) is is 2-dimensional uniform random variable defined over the triangle with corners $(0, 0)$, $(1, 0)$ and $(0, 1)$. Prove that X and Y cannot be independent.