# Study sheet for Midterm 2, Math 3770

November 2, 2008

- You will be allowed to bring a basic calculator to the exam. If you bring a scientific calculator, make sure that it does not have any programmable features.

- Know all the material from the first exam – i.e. sections 1 through 3.2 of your book. And, know the material from sections 3.2 to 7.1. The remaining items below are meant to emphasize what portions of this I regard as most important for the exam.

- Know the distributions of the following basic random variables: binomial, uniform, Poisson, exponential, and normal. In addition, know that the chi-squared random variable with $k$ degrees of freedom has the same distribution as

$$X_1^2 + \cdots + X_k^2,$$

  where the $X_i$ are independent, $N(0,1)$ random variables. I will give you the remaining distributions on the exam.

- Know how to compute the expected value of some basic random variables. For example, I may give you the pdf of a r.v., and ask you to compute its expected value. Also, know how to compute the expectation of a function of a random variable.

- Know basic properties of expectation, such as linearity: If $X_1, ..., X_k$ are random variables (which may or may not be independent), then

$$E(a_1 X_1 + \cdots + a_k X_k) = a_1 E(X_1) + \cdots + a_k E(X_k).$$

- Know basic properties of variance: If $X_1, ..., X_k$ are **pairwise independent**, then

$$V(a_1X_1 + \cdots + a_kX_k) \;=\; a_1^2V(X_1) + \cdots + a_k^2V(X_k).$$

  Know also the shortcut

$$V(X) \;=\; E(X^2) - E(X)^2.$$

- Know the definition of standard deviation, and know that sample variance and sample standard deviation is slightly different from the one for r.v.'s.

- Know what a binomial experiment entails (you will not need to know its precise definition, but you should be able to recognize from the statement of a problem, whether or not you are dealing with a binomial r.v.).

- Know what a hypergeometric random variable is. You should be able to derive its pdf via some simple counting.

- I will not hold you responsible for knowing the pdf for a negative binomial r.v., but you should be able to derive it based on some easy counting.

- Know roughly what a Poisson process is, and be able to do some basic manipulations of Poisson r.v.'s, such as compute its expected value. Know the basic fact that if $X$ and $Y$ are independent Poisson r.v.'s with parameters $\lambda_1$ and $\lambda_2$, respectively, then $X + Y$ is Poisson with parameter $\lambda_1 + \lambda_2$. Know how to approximate binomial r.v.'s using Poissons: If $X$ is binomial with parameter $p \sim \lambda/n$, and $n$ large, then $X$ has roughly a Poisson distribution with parameter $\lambda$. Be able to work some computational problems using Poisson r.v.'s, such as in the lecture note posted on course webpage.

- Know how to compute the cumulative distribution of a function of a r.v. Usually, this works as follows: Let $X$ be a r.v., and let $h(X)$ be some function of $X$. Typically, one first computes the cdf of $h(X)$ – i.e. $P(h(X) \leq x)$ – and then one takes its derivative. Of course, there are short-cuts, but let's not bother with these.

- Know the definition of percentile and median of a random variable (distribution).

- Know how to prove that $\int_{-\infty}^{\infty} e^{-t^2/2} dt = \sqrt{2\pi}$.

- Know how to use a normal distribution table to find values of $\phi(x) =$ cdf for $N(0,1)$. Know how to transform an $N(\mu, \sigma^2)$ r.v. into one that is $N(0,1)$, so that lookup tables may be applied – e.g. if $X = N(\mu, \sigma^2)$, then $(X - \mu)/\sigma = N(0,1)$.

- If I give you the definition of the gamma function $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ on the exam, be able to prove that $\Gamma(n) = (n-1)!$ for $n \geq 1$ an integer (by convention, $0! = 1$). Know some basic facts about the gamma function, such as that $\Gamma(1/2) = \sqrt{\pi}$ and $\Gamma(n) = (n-1)!$.

- Know how to work some problems about the chi-squared distribution, especially the submarine example from class.

- Know the basic definitions of pdf's for multiple random variables – i.e. know what the joint pdf $f(x_1, ..., x_n)$ means, and know what the marginals $f_{X_i}(x_i)$ mean (we only worked problems for the case $n = 2$ in class). Know how to compute conditional pdf's. Know what it means for a collection of r.v.'s $(X_1, ..., X_n)$ to be independent.

- Know the definition of covariance:

  $$\mathrm{Cov}(X, Y) \ = \ \mathrm{E}((X - \mu_X)(Y - \mu_Y)).$$

  Know the short-cut that this equals $\mathrm{E}(XY) - \mu_X \mu_Y$. Know that if $X$ and $Y$ are independent, then $\mathrm{Cov}(X, Y) = 0$, **but that the converse is not true**. Be able to present a simple example that disproves the converse. Know basic properties of the correlation coefficient – its definition, and the fact that

  $$\mathrm{Corr}(aX + b, cY + d) \ = \ \pm \mathrm{Corr}(X, Y).$$

  Know that if $Y$ is a non-trivial linear function of $X$, then $\mathrm{Corr}(X, Y)$ is $\pm 1$, and vice versa.

- Know the Central Limit Theorem, and be able to solve some problems about it. Consult the notes on the course webpage. Also, a few examples were presented in class that are not in these notes (like the portfolio problem.)

- Know the basics of estimators, such as what MVE and MLE estimators are. Know how to find a MLE estimator for some hidden parameters of some given r.v. Know what it means for an estimator to be unbiased, and know that the sample variance is unbiased in the case where $X$ is $N(\mu, \sigma^2)$; YET, the MLE estimator for the $\sigma^2$ turns out to be biased. Know that for large sample sizes the MLE is close to being unbiased, and has nearly the smallest variance that is possible. Sometimes, calculus is of little help in computing an MLE; however, know how to find the MLE in a few such cases, such as when $X$ is uniform over $[0, \theta]$, where $\theta$ is the hidden parameter to be estimated.

- Know what moments are, and know how to find them for a given r.v. Know how to compute the moment generating function of a r.v., and how to use it to compute some moments. Know how to find moment estimators for some hidden parameters of a r.v. $X$.

- Know what a confidence interval is. Know how to compute one in the case $X = N(\mu, \sigma^2)$, where $\sigma$ is given to you. Know how to start with a given level of confidence (like 95%, say), and work out how large $n$, the sample size, needs to be in order for the confidence interval to have width $w$.