Math 4107, Midterm 2, Fall 2009

November 12, 2009

1. Define the following terms.
  a. Ring (list the axioms).
  b. Centralizer of an element g in a group G.
  c. State the three Sylow theorems.
  d. Cayley's Theorem.
  e. Alternating group.

2. Express the following permutation in disjoint cycle form (composition here works from right-to-left):

$$(1\ 7\ 5\ 6\ 3)(2\ 1\ 4)(4\ 7\ 5\ 2\ 3)\ \in\ S_7.$$

3. Determine the center Z of S_n for $n \geq 2$, and prove your answer.

4. Suppose G is a group of order pq, $p < q$ both prime, such that G acts non-trivially on a set X having size q. Prove that G is abelian.

5. Determine the number of abelian groups of order $3^4 \cdot 5^2$, and list one example from each isomorphism class.