Additive Combinatorics, Math 8803/4803

January 9, 2011

- **Instructor:** Ernie Croot
- **Office:** 103 Skiles.
- **Office Hours:** TBA
- **Meeting Time and Place:** MWF 1:05 - 1:55, Skiles 154.
- **Textbook:** I will be using the Tao-Vu book *Additive Combinatorics*, 2nd edition (ISBN-10 0521136563, ISBN-13 978-0521136563) to assign homework and to occasionally lecture from. I also plan to use several other sources, and I will post links or pdf files to notes when I do so.
- **Grade:** Basically, if you show up for most of the classes and turn in the homeworks (and at least make a good attempt) I will give you an A. There will be no in-class exams in this course. Near the end of the semester I *may* ask people to present lectures on a topic of their choice (within limits).
- **Content:** Mainly, I will focus on tools rather than big theorems. (In many cases if one understands the tools, and roughly how they are used in the proof of a big theorem, one can reproduce the proof oneself anyways.)

  In the beginning I plan to focus on elementary properties of sumsets, proving such results (tools) as the Cauchy-Davenport Theorem; the Balog-Szemeredi-Gowers Theorem; a structure theorem due to myself, Ruzsa and Schoen about sumsets; various properties of the "energy of sumsets"; the Brun Sieve and Rosser's Sieve; Schnirelmann's theorem and applications; the Plunnecke-Ruzsa Inequality; various "Ruzsa-type" inequalities; Behrend's construction; Szemeredi's elementary proof of Roth's Theorem; Solymosi's sum-product inequality (for the complex numbers); the Bourgain-Katz-Tao sum-product inequality (with applications); and possibly several more.

Then, I will switch gears somewhat and start talking about Fourier methods, and in particular will present: Roth's theorem on three-term progressions; the Szemeredi and Heath-Brown refinement; the Littlewood-Offord Inequality and various results about subset sums; and Ruzsa's proof of Freiman's theorem.

If time permits, I will discuss higher-order Fourier methods as developed by Gowers (and possibly ergodic methods), including an overview of his proof of Szemeredi's theorem for four-term arithmetic progressions.