

HW set 2, Problem 3

March 4, 2011

Definition. We say that a function $h : \mathbb{Z}_N \rightarrow \mathbb{C}$ is ε -quasirandom if

$$\|h * h\|_2^2 = N^{-1} \sum_{\chi \in \hat{\mathbb{Z}}_N} |\hat{h}(\chi)|^4 \leq \varepsilon N^3.$$

Show the following: Suppose that $f, g : \mathbb{Z}_N \rightarrow \mathbb{C}$, with $\|f\|_\infty, \|g\|_\infty \leq 1$. Further, suppose that $f - g$ is ε -quasirandom. Show that this implies that $f * f - g * g$ is $2\varepsilon^{1/2}$ -quasirandom.