

TRANSVERSE UNIVERSAL LINKS

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ABSTRACT. We show that there exists a transverse link in the standard contact structures on the 3–sphere such that all contact 3–manifolds are contact branched covers over this transverse link.

1. INTRODUCTION

In 1982, William Thurston showed there was a six component link in the 3–sphere such that any closed smooth oriented 3–manifold is the branched cover over the 3–sphere with branch locus this link [20]. He called such a link universal. Later Hilden, Lozano, and Montesinos showed there was a universal knot in the 3–sphere [10, 11]. Since then there has been several papers showing certain knots and links are universal or not universal. For example the figure eight knot, Whitehead link, and Borromean rings are all universal [13, 12].

In 2002, Emmanuel Giroux showed that any contact 3–manifold is a 3–fold simple branched cover of the 3–sphere with the standard contact structure (S^3, ξ_{std}) and branch locus a transverse link [7]. This is a contact strengthening of the Hilden-Montesinos Theorem [9, 16] for smooth 3–manifolds. These contact constructions are useful for constructing open books for some contact 3–manifolds and embedding contact 3–manifolds in the standard contact structure on the 5–sphere [5].

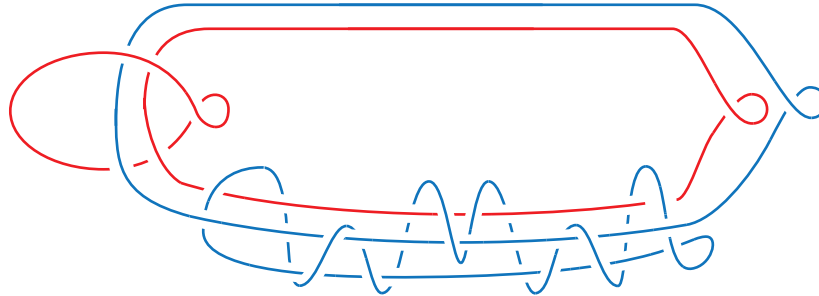


FIGURE 1. The universal transverse link in Theorem 1.1.

Given this work it is natural to wonder if there is a universal transverse link in (S^3, ξ_{std}) . That is, a transverse link $L \subseteq (S^3, \xi_{std})$ such that *any* contact 3–manifold is the contact branched cover of (S^3, ξ_{std}) along L . This question was first considered by M. Casey in [3] where she showed that no transverse knot in the knot type of the figure eight can be universal and that many covers of the Whitehead link and Borromean rings yielded only overtwisted contact structures, raising doubts as to whether or not there exist universal transverse links. The main result in the present work is that they exist:

Theorem 1.1. *Any contact 3–manifold (M, ξ) can be realized as a contact branched cover of (S^3, ξ_{std}) branched along the transverse link L shown in Figure 1.*

The proof presented here directly adapts the argument for the existence of topologically universal links in [10] to the contact case. It is interesting to note that similar constructions

of universal links that lead to universal knots cannot be easily adapted to the contact setting, leaving open the natural question

- (a) Does there exist a transverse universal knot $K \subseteq (S^3, \xi_{std})$?

The Figure-8 knot is a smooth universal knot, but it cannot be a contact universal knot after the results from M. Casey's thesis [3]. This leads to the following problem:

- (b) Find sufficient conditions for a smooth universal knot or link to admit transverse representatives which are contact universal.

In line with the first question, we would also like to know the answer to the following question:

- (c) Let (S^3, ξ_{ot}) be an overtwisted contact structure. Does there exist a transverse knot $K \subseteq (S^3, \xi_{ot})$ such that the contact branched cover of (S^3, ξ_{ot}) along K is contactomorphic to (S^3, ξ_{std}) ?

It is possible to produce branched covers of overtwisted structures on S^3 which are tight *cf.* [3], yet it is apparent that such K in the question — should it exist — cannot be a smooth unknot.

Finally, a natural continuation of Theorem 1.1, in line with [15, 19], would be to answer the following question:

- (d) Does there exist a symplectic surface $S \subseteq (D^4, \lambda_{std})$ such that any Weinstein 4-fold (W, λ) is a branched cover of (D^4, λ_{std}) along S ?

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2. BACKGROUND

In this section we recall some necessary definitions concerning transverse knots and branched covers. In the last subsection we show that many standard modifications of the branch locus for branched covering maps that are known in the topological setting also hold in the contact geometric setting as well.

2.1. Transverse knots. For more details on transverse knots the reader is referred to [4]. Here we briefly review the aspects that will be relevant for our main result.

We will consider our knots in $(\mathbb{R}^3, \xi_{std}) \subset (S^3, \xi_{std})$ where on \mathbb{R}^3 we have the standard contact structure

$$\xi_{std} = \ker(dz - y dx),$$

where (x, y, z) are Cartesian coordinates on \mathbb{R}^3 . A knot K is *transverse* if K is transverse to ξ_{std} at every point of K . Since ξ_{std} is co-oriented by the contact form and \mathbb{R}^3 is oriented, K will have an orientation induced on it so that K and ξ_{std} intersect positively.

We will study transverse knots via their *front projection*, that is, via the image of K under the projection $\pi : \mathbb{R}^3 \rightarrow \mathbb{R}^2 : (x, y, z) \mapsto (x, z)$. Notice that being transverse to ξ_{std} implies that the y -coordinate of K satisfies

$$y < \frac{dz}{dx},$$

where we think of the page as the xz -plane so that the positive y -axis points into the page. It is easy to see that any diagram for an oriented knot in the xz -plane determines a transverse knot up to isotopy through transverse knots as long as no portion of the knot is as shown in Figure 2. Moreover, Type II and III Reidemeister moves are allowed in front diagrams as long as no portion of the move contains a forbidden diagram from Figure 2.

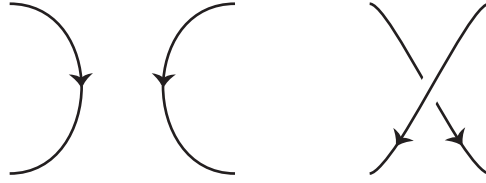


FIGURE 2. Forbidden portions of the front diagram of a transverse knot.

In addition to the *front projections*, there is a different presentation of transverse knots that will be useful for us as well. Consider $S^3 = \{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^2 = 1\}$ as the unit sphere in \mathbb{C}^2 with the standard contact structure $\xi_{std} = TS^3 \cap i(TS^3)$ given as the set of complex tangencies to S^3 . The link $H = \{z_1 = 0\} \cup \{z_2 = 0\}$ is a Hopf link with each component being an unknot with self-linking number -1 . It is easy to check that the contact structure on the complement $S^3 - H = T^2 \times (0, 1)$ is given by

$$\xi_{std} = \ker \left(\cos \left(\frac{\pi}{2} t \right) d\phi + \sin \left(\frac{\pi}{2} t \right) d\theta \right),$$

where (ϕ, θ) are angular coordinates on $T^2 = S^1 \times S^1$ and t is the coordinate on $(0, 1)$. We can now consider transverse knots in $T^2 \times (0, 1)$ and their *front projection* will be obtained by projecting out the t -coordinate. Once again a transverse knot can be recovered from its front projection. The only real difference with the situation described above is that the tangent vector to the projection cannot have both ϕ and θ component negative.

A closed transverse braid in \mathbb{R}^3 can always be assumed to lie in the thickened torus $T^2 \times (0, 1)$ and such braids are in one-to-one correspondence curves in $T^2 \times (0, 1)$ whose oriented tangents always have positive θ -coordinate [1].

2.2. Contact branched covers. A *branched covering map* is a smooth map $p : M \rightarrow Y$ between smooth 3-manifolds such that there is a link $L \subseteq Y$, called the *branch locus*, such that p restricted to $M - p^{-1}(L)$ is a covering map from $M - p^{-1}(L)$ to $Y - L$ and each component of $p^{-1}(L)$ has a neighborhood $N = S^1 \times D^2$ such that $p(N) = S^1 \times D^2$ and in these coordinates p is of the form $(\phi, z) \mapsto (n\phi, z^m)$ for some integers n and m where ϕ is the angular coordinate on S^1 and z is the complex coordinate on D^2 thought of as the unit disk in \mathbb{C} . We say the component has *order* m and we say p is *ramified* at the component if $m > 1$. An branched covering map of degree k is called *simple* if the pre-image of any point has size either k or $k - 1$.

Given a contact submanifold, it is quite easy to construct contact structures on branched covering spaces from a contact structure on the base space.

Theorem 2.1 (Geiges 1997, [6]; Öztürk and Niederkrüger 2007, [18]). *Let $p : M \rightarrow Y$ be a branched covering map with branch locus L . Given a contact structure ξ on Y with contact form α such that L is transverse to ξ , then $p^*\alpha$ may be deformed by an arbitrarily small amount near $p^{-1}(L)$ to give a contact form defining a unique, up to contact isotopy, contact structure ξ_L on M .*

The starting point for our main result is Giroux’s proof that every contact 3-manifold can be constructed as a branched cover over the standard contact structure on S^3 .

Theorem 2.2 (Giroux 2002, [7]). *Given a contact structure ξ on a 3-manifold M , there is a 3-fold simple branched covering map $p : M \rightarrow S^3$ such that ξ is the contact structure induced on M by p , the standard contact structure on S^3 , and some transverse realization of the branch locus of p .*

For each contact 3-manifold (M, ξ) , Theorem 2.2 asserts the existence of a transverse link $L(M, \xi) \subseteq (S^3, \xi_{std})$, depending on (M, ξ) , such that (M, ξ) is the contact branched cover of

(S^3, ξ_{std}) along $L(M, \xi)$. Our main result Theorem 1.1 shows that the transverse link $L(M, \xi)$ can be chosen to be independent of (M, ξ) , thus encoding all the complexity of the contact structure on (M, ξ) in the combinatorics of the deck transformation group for coverings of $S^3 \setminus L$.

Let us briefly recall the argument for Theorem 2.2. E. Giroux first shows that any contact 3-manifold is supported by an open book decomposition, whose binding can be assumed to be connected. Building on work of J.S. Birman [2] and D.L. Goldsmith [8], J.M. Montesinos and H.R. Morton showed in [17] that any open book decomposition on a smooth 3-manifold M with connected binding is pulled back from the open book decomposition of S^3 with disk page by a 3-fold branched cover map $p : M \rightarrow S^3$ whose branch locus is a braid. Since a braid is naturally a transverse knot the branched cover can be taken to be a contact branched cover and it is easy to see that the induced contact structure on the covering space is supported by the original open book, thus concluding the statement.

We end this subsection with two simple observations. The first follows immediately from the definition of contact branched cover.

Lemma 2.3. *Suppose $p : (M', \xi') \rightarrow (M, \xi)$ is a contact branched covering map with branch set $B \subset M$. Let L be a link in M disjoint from B and $L' = p^{-1}(L)$. If $p' : (M'', \xi'') \rightarrow (M', \xi')$ is a contact branched covering map with branch set L' , then $p' \circ p : (M'', \xi'') \rightarrow (M, \xi)$ is a contact branched covering map with branch set $B \cup L$. \square*

The second fact is also standard in contact topology:

Lemma 2.4. *The contact manifold obtained from any branched cover of (S^3, ξ_{std}) branched along a transverse unknot with self-linking number -1 is contactomorphic to (S^3, ξ_{std}) .*

Proof. It is clear that smoothly the branched cover of S^3 branched over the unknot is diffeomorphic to S^3 . Moreover if U is the transverse unknot in the standard contact structure on S^3 with self-linking number -1 then $S^3 - U = S^1 \times \mathbb{R}^2$ with the contact structure $\xi = \ker(d\phi + r^2 d\theta)$, where ϕ is the angular coordinate on S^1 and (r, θ) are polar coordinates on \mathbb{R}^2 . Now any finite covering space of $(S^1 \times \mathbb{R}^2, \xi)$ is contactomorphic to $(S^1 \times \mathbb{R}^2, \xi)$, from which the result follows. \square

The hypothesis on the self-linking number in Lemma 2.4 is meaningful, since branched covers along stabilized transverse knots are overtwisted.

2.3. Monodromy representations. A covering map $p : M \rightarrow Y$, for M connected, is determined by its monodromy representation. Let $x_0 \in Y$ be a fixed base point and label the points in $p^{-1}(x_0) = \{x_1, \dots, x_n\}$. Then, given a loop γ in Y based at x_0 we can define the element σ_γ of the symmetric group S_n by $\sigma_\gamma(i)$ is the index on $\tilde{\gamma}(1)$ where $\tilde{\gamma}$ is the lift of γ based at x_i . This defines a transitive representation of the fundamental group

$$r : \pi_1(Y) \rightarrow S_n = \text{Aut}(p^{-1}(x_0)).$$

Moreover, given such a representation the subgroup $G = \{g \in \pi_1(Y) : r(g)(1) = 1\}$ gives a covering space M that corresponds to the representation.

One may similarly prove that a branched covering map $p : M \rightarrow Y$ with branch locus L is determined by a representation $\pi_1(Y - L) \rightarrow S_n$, since any covering of $Y - L$ may be uniquely extended to a branched covering map of Y thanks to the local model near the branch locus.

In the case that Y is smoothly S^3 , the fundamental group $\pi_1(Y - L)$ is well-known to be generated by meridians to L . In particular, given a diagram for L we have the Wirtinger presentation for $\pi_1(Y - L)$ with generators x_i corresponding to the strands in the diagram and relations coming from the crossings: $x_i = x_k^{-1} x_j x_k$, respectively $x_i = x_k x_j x_k^{-1}$, at a right handed, respectively left handed, crossing, where strand k goes over incoming strand j and

outgoing strand i . Thus a branched covering map over S^3 with branch set L is determined by labeling a diagram of L with elements of S_n that satisfy the required relations at the crossings.

It is easy to see if a branched covering map is simple using the monodromy representation: one just needs the image of the representation to consist only of transpositions, for higher length cycles correspond to more than two points coming together.

We now discuss some modifications one can make to the branch locus of a branched covering map without affecting the induced contact covering space.

Lemma 2.5. *Let L be a transverse link in a contact 3-manifold (Y, ξ) and $p: M \rightarrow Y$ be a branched covering map with branch locus L inducing the contact structure ξ' on M . If part of a diagram for L is as shown on one side of a row in Figure 3 then replacing that portion of L with the other diagram shown in that row will result in a new contact branched covering of Y that still yields the same contact manifold (M, ξ') .*

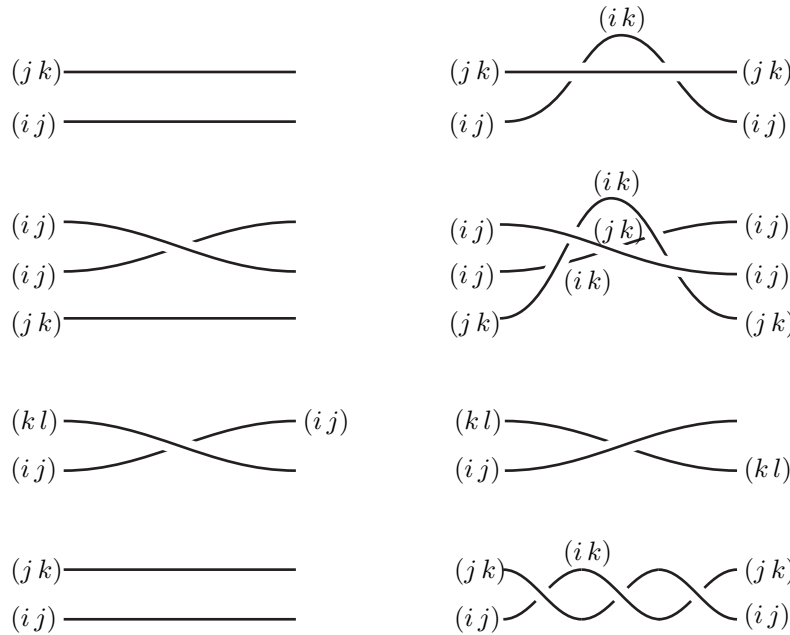


FIGURE 3. Replacing a portion of the branch locus of a simple cover with one of the figures in a row with the other does not change the manifold or contact structure described by the contact branched cover. All strands are oriented from left to right.

Proof. The changes to the diagram depicted in the first two rows of Figure 3 correspond to contact isotopies of the underlying branch locus. This induces an isotopy of the branch cover manifold, thus does not change the branched cover or the contact structure.

The change in the fourth row was proven to leave the contact branched cover unchanged in [3] and can easily be seen by observing that the branched cover of the ball containing either branched loci is simply a ball and the contact structure on it is tight. See [3] for details.

That the change in the third row does not affect the contact branched cover follows exactly as for the fourth row. That is, the pre-image by the covering map of a ball containing the diagram on either side is a union of balls and the contact structure on each ball is tight. \square

There is a second simple modification to the transverse branched locus which preserves the contact isotopy type of the branched cover, it adds a standard transverse unknot to the branch locus at the cost of increasing the degree of the branched cover:

Lemma 2.6. *Let L be a transverse link in a contact 3-manifold (Y, ξ) and $p: M \rightarrow Y$ be an n -fold branched covering map with branch locus L inducing the contact structure ξ' on M . Let L' be L together with a maximal self-linking unknot U that is separated from L by a sphere.*

The monodromy data for L' given by using the monodromy data on L and labelling the meridian of U by $(n, n+1)$ defines an $(n+1)$ -fold branch covering map from M to Y that induces the same contact structure ξ' on M .

Proof. We begin with a simple observation. Let $K = K_0 \cup K_1$ be a link with monodromy data for an n -fold branched covering map, such that there is a sphere S that separates K_0 and K_1 . We can assume that S is a convex sphere and cut S^3 into two contact 3-balls B_0 and B_1 along S so that B_i contains K_i . Then cap off B_i by a tight contact 3-ball C_i to get a tight contact 3-sphere S_i^3 that contains K_i . Let $p_i: M_i \rightarrow S_i^3$ be the n -fold branched covering map determined by K_i and its monodromy data. Note that the pre-image $p_i^{-1}(C_i)$ consists of n distinct 3-balls. Moreover if $p: Y \rightarrow S^3$ is the branched covering map corresponding to K then $Y \setminus p^{-1}(S)$ is contactomorphic to $(Y_0 \setminus p_0^{-1}(C_0)) \cup (Y_1 \setminus p_1^{-1}(C_1))$. In consequence, the contact manifold Y is the contact n -fold, possibly internal, connect sum of Y_0 and Y_1 .

Now given the link $L' = L \cup U$ in the lemma notice that $Y_0 = Y \cup S^3$ since Y_0 will be an $(n+1)$ -fold cover of S^3 branched along L , where the monodromy data for L only has labels between 1 and n , so the cover is disconnected and the $(n+1)$ -sheet is disjoint from the rest. Similarly Y_1 is the union of n copies of 3-spheres by Lemma 2.4. It then follows that the $(n+1)$ -fold contact connected sum of Y_0 and Y_1 results in a manifold contactomorphic to Y . \square

3. TRANSVERSE UNIVERSAL LINKS

In this section we prove Theorem 1.1 by showing the link L in Figure 1 is a universal transverse link $L \subseteq (S^3, \xi_{std})$. The proof will follow from two following lemmas.

Lemma 3.1. *Let L' be the link shown in Figure 4. There is a branched cover*

$$p: (S^3, \xi_{std}) \rightarrow (S^3, \xi_{std}),$$

branched along the Hopf link discussed at the end of Section 2.1 such that the link $L_{m,n}$ shown in Figure 5 is a sub-link of $p^{-1}(L')$.

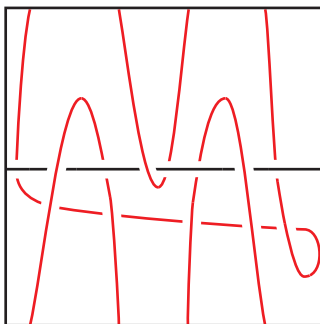


FIGURE 4. The front projection of the link L' in $T^2 \times (0, 1) = S^3 - H$.

Lemma 3.2. *Any contact 3-manifold (Y, ξ) can be realized as a contact branched cover of (S^3, ξ_{std}) branched along the transverse link $L_{n,m}$ shown in Figure 5, for some n and m .*

Lemmas 3.1 and 3.2, which will be momentarily proven, suffice to conclude our main result:

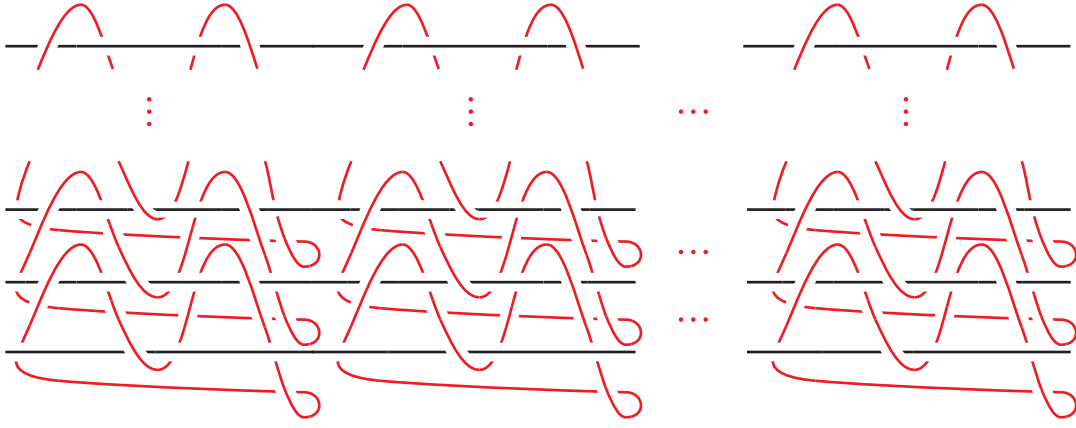


FIGURE 5. The transverse link $L_{m,n}$ in Lemma 3.1. There are n horizontal lines that are closed into a trivial braid and there are m vertical rows of clasping unknots. Lemma 3.2 states that for any contact manifold (Y, ξ) there is a choice of $n, m \in \mathbb{N}$ such that (Y, ξ) is the contact branched cover of (S^3, ξ_{std}) over this transverse link.

Proof of Theorem 1.1. Let (M, ξ) be any given contact 3-manifold, we can apply Lemma 3.2 to construct a branched covering map $p' : (M, \xi) \rightarrow (S^3, \xi_{std})$ with branched set $L_{n,m}$. By Lemma 3.1 there is also a branched covering map $p : (S^3, \xi_{std}) \rightarrow (S^3, \xi_{std})$ with branched set the red Hopf link H depicted in Figure 1 such that $L_{n,m}$ is a sub-link of $p^{-1}(L')$, where $L' \subseteq T^2 \times (0, 1) \simeq S^3 \setminus H$ is the link presented in Figure 4. Then Lemma 2.3 implies that the composition $p' \circ p : (Y, \xi) \rightarrow (S^3, \xi_{std})$ is a contact branched covering map with branch set the transverse link $L' \subseteq T^2 \times (0, 1) = S^3 - H$ union the transverse Hopf link $H \subset S^3$.

In order to conclude Theorem 1.1 it suffices to identify the link in Figure 4 with the blue sub-link in Figure 1. This is tantamount to identifying the contact structures

$$\xi_0 = \ker\{2dz - xdy + ydx\}, \quad \xi_{std} = \ker\{dz - ydx\},$$

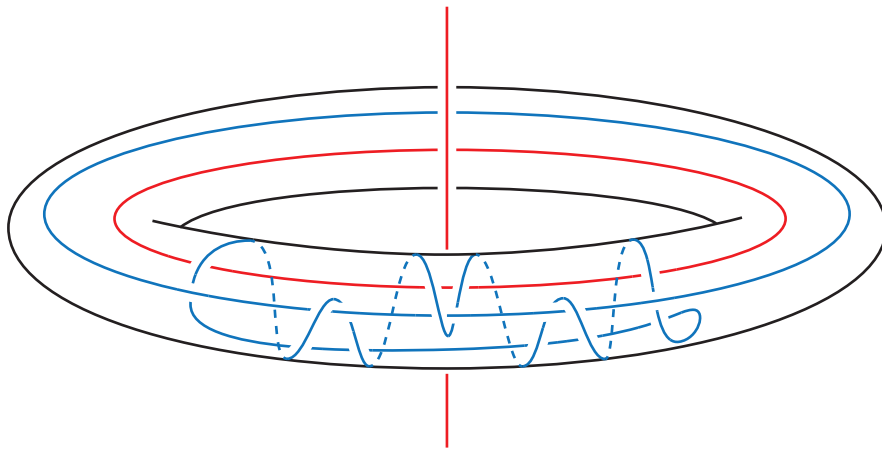
the former being the rotationally invariant contact structure on \mathbb{R}^3 and the latter the standard contact structure. The contactomorphism from the first to the second is explicitly given by the map $\varphi(x, y, z) = (x, 2y, xy + z)$ extended to S^3 by fixing the point at infinity.

Now we consider the 2-component transverse link in Figure 4 with $T^2 \times (0, 1)$ embedded in (S^3, ξ_0) (see the end of Section 2.1 for this embedding), as depicted in the left of Figure 6 with the components of L' shown in blue and the Hopf link H shown in red. For any given $\varepsilon \in \mathbb{R}^+$, the non-horizontal blue component can be confined inside the region $|x| \leq \varepsilon$, and so the contactomorphism $\varphi : (S^3, \xi_0) \rightarrow (S^3, \xi_{std})$ can be assumed to be C^∞ -close to the identity in that region. It is a simple exercise to show that the other blue component is, up to contact isotopy, sent to itself with an added loop when considered in the front projection for $(\mathbb{R}^3, \xi_{std})$, see for instance [14, Figure 6]. In consequence, adding the red Hopf link to this link gives the link in Figure 1, as desired.

□

Let us now prove the two Lemmas 3.1 and 3.2.

Proof of Lemma 3.1. Let $p_{m,n} : (T^2 \times (0, 1)) \rightarrow (T^2 \times (0, 1))$ be the covering map that unwinds the θ -circle m times and the ϕ -circle n times, where we are using the coordinates from Section 2.1. This covering map smoothly extends to a branched covering map $p : S^3 \rightarrow S^3$. Moreover, since $(p_{m,n})^*(\xi_{std})$ is contactomorphic to ξ_{std} , it is clear the contact structure

FIGURE 6. The link L' in ξ_0

induced on S^3 by this branched covering map is the standard contact structure. By construction, the link $L_{m,n}$ in Figure 5 is a sub-link of the pre-image $p^{-1}(L')$ of L' via this explicit branched cover. \square

Proof of Lemma 3.2. Given a contact manifold (M, ξ) , we apply Theorem 2.2 to construct a 3-fold simple branched cover from M onto (S^3, ξ_{std}) with branch locus a transverse braid B , such that ξ is induced from the covering. Note that since the covering map is simple, the labels on the strands of B are all transpositions. We will now modify the transverse braid B without changing the contact isotopy type of the branched cover (M, ξ) until the B has the desired form depicted in Figure 5. *Step 1: Change B so that at each crossing the*

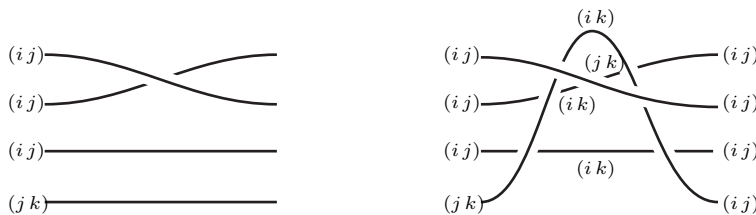


FIGURE 7. Arranging all crossings to have multiple labelings on the strands.

strands have three distinct labels. Thinking of B as the closure of a braid we assume B has n strands. We will think of a diagram for B as n horizontal strands the xz -plane with twists at distinct x values, i.e. presented using standard generators of the braid group. Each of the n different strands at any x value must be labelled with at least two different transpositions in the symmetric group. If there is a single x value where there is just one then all strands in the braid will be labelled by this transposition and the covering is not a 3-folding covering, but just a cyclic 2-fold covering.

Now, let us suppose c is a crossing in the transverse braid B . The strands at c will either be labelled with three distinct transpositions or just one. In the former case, there is nothing to do, whereas in the latter case some other strand of the braid with the same x -value as the crossing must be labelled with a different transposition. Taking the strand with a different permutation that is closest to the crossing we can push it past the crossing as shown in Figure 7. This is done by a sequence of moves shown in the first two rows of Figure 3 and results in a braid with distinct labels at each crossing.

Step 2: Change B so that it has only positive crossings and they occur in pairs. We can apply the move shown in row four of Figure 3 to each negative crossing in order to change it into a pair of two positive crossings. Note that we also need to use the move in row one to

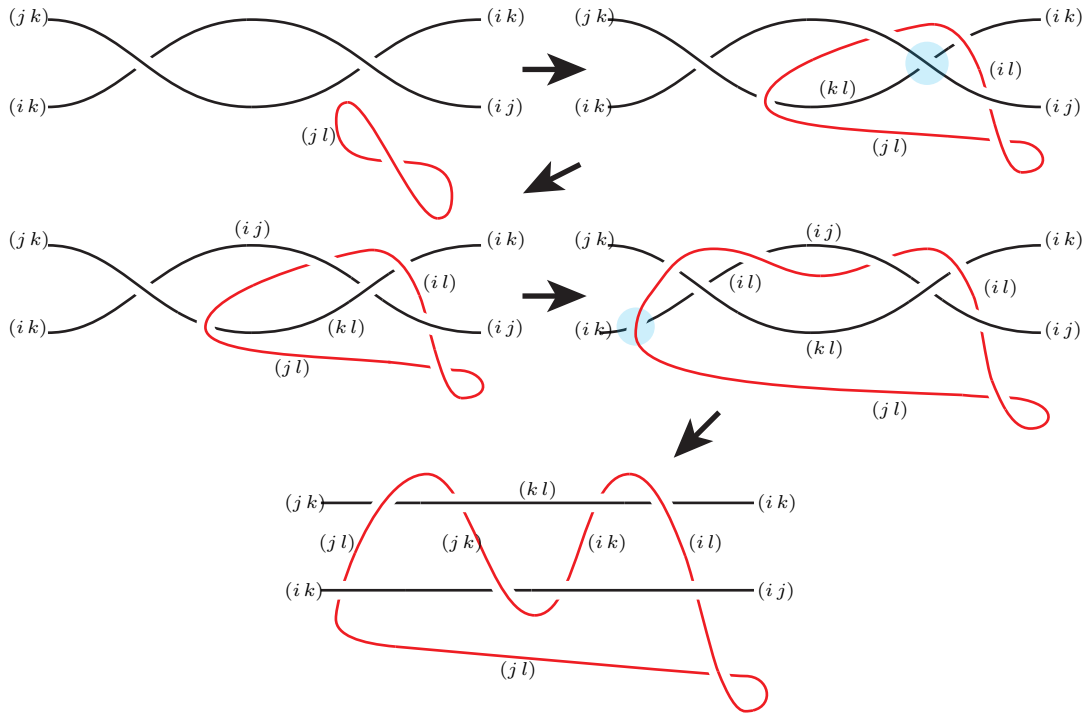


FIGURE 8. Adding an unknotted component to the branch locus.

remove adjacent positive and negative crossings. For each unpaired positive crossing we can just change them into two paired positive crossings.

Step 3: Change B so that it is the trivial braid with linking unknots as in Figure 5. For each pair of positive crossings we can use Lemma 2.6 to introduce an unknot with label $(j\ l)$, where when considering this crossing we already have an $(l - 1)$ -fold branched cover and j is chosen as shown in Figure 8. Now Figure 8 shows a sequence of isotopies of the unknot and applications of the move in row three of Figure 3 which manages to remove the pair of crossings at the expense of inserting a linked unknot. We will call such an unknot a *clasping unknot*.

If the braid after Step 2 had k pairs of crossings, then after Step 3, there are k clasping unknots and no clasping unknot is above another. Then adding $(n - 2)$ new clasping unknots labelled with the identity permutation to the strands above and below each existing clasping unknot yields the link in Figure 5. Lemma 3.2 is now proven once we note that adding a knot to the branch locus whose strands are labelled with the identity permutation does not change the contact branched covering. \square

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