1.(a) Let $\mathbf{x_1} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{x_2} = \begin{pmatrix} 2 \\ 1 \\ 3 \\ 1 \end{pmatrix}$ and $\mathbf{x_3} = \begin{pmatrix} 3 \\ 1 \\ 5 \\ 1 \end{pmatrix}$ and $S = \text{Span}\{\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}\}$. Find an orthonormal basis for S.

(b) Find the expansion of $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 0 \end{pmatrix}$ in terms of this basis

(c) Find the orthogonal projection onto S.

2.(a) Let $\mathbf{x_1} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{x_2} = \begin{pmatrix} 2 \\ 1 \\ 3 \\ 1 \end{pmatrix}$ and $\mathbf{x_3} = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ and $S = \text{Span}\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$.

(a) Find the orthogonal complement of S. It is a subspace of what space?

(b) Let $A = [\mathbf{x_1}, \mathbf{x_1}, \mathbf{x_1}]$. Decompose $A = QR$ where the columns of Q are an orthonormal basis for S, and R is an upper triangular matrix with positive diagonal entries.

3 Let $\mathbf{x_1} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{x_1} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix}$. Find,

(a) $|\mathbf{x_1}|$, $|\mathbf{x_1}|$.

(b) The cos of the angle between $\mathbf{x_1}$ and $\mathbf{x_2}$.

(c) $|\mathbf{x_1} + \mathbf{x_2}|$.

4 Consider the line L in R^3 passing through the origin and the point $(1, 2, 1)$. Let $\bar{x} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$.

(a) Find $\bar{x}_{||}$ the projection of $\bar{x}$ on the line L.

(b) Find $\bar{x}_{\perp}$.

(c) Find the matrix representation of the orthogonal projection onto L.

5(a) If $S = \{\bar{u}_1, \ldots \bar{u}_k\}$ is a set of nonzero orthogonal vectors show that this is a lineraly independent set.

5(b) If A is an $n \times m$ matrix show $(\text{Null}(A))^{\perp} = \text{col}A^T$