110.201 July 5, 1996
Linear Algebra
Summer 1996, JHU

FINAL EXAM

Time: 180min

1. Let A and B be invertible matrices. Show that AB is also invertible and prove: $(AB)^{-1} = B^{-1}A^{-1}$.

2. Let A be a 2×2 matrix. Show that A and A^T have the same eigenvalues. Prove this fact for a general $n \times n$ matrix.

3. Let $D : P_3 \to P_2$ be the linear transformation given by $D(P(x)) = P'(x)$. Find the matrix of D with respect to the standard bases. Show that D is not one-to-one. What is the kernel of D?

4. Find the equation of the circle that passes through the three points (1,7), (6,2), and (4,6). (Recall that the equation of a circle in R^2 can be expresses in the form $c_1(x^2 + y^2) + c_2x + c_3y + c_4 = 0$.)

5. Find a basis for the span of $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$. Express each element as a linear combination of the basis vectors.

$$\mathbf{v_1} = (1,0,1,1), \mathbf{v_2} = (-3,3,7,1), \mathbf{v_3} = (-1,3,9,3), \mathbf{v_4} = (-5,3,5,-1)$$

6. Find the quadratic polynomial that best fits the four points $(2,0)$, $(3,-10)$, $(5,-48)$, and $(6,-76)$.

7. Translate and rotate the coordinate axis, if necessary, to put the conic section below in standard position. Name the conic and give its equation in the final coordinate system.

$$9x^2 - 4xy + 6y^2 - 10x - 20y = 5$$

Turn Over →

8. Let l be the line in R^3 spanned by $v = (1, 1, 1)$ and $T : R^3 \to R^3$ be rotation around l by an angle of θ. What is the matrix of T with respect to the standard basis? Can $[T]$ be orthogonalized? (Hint: Find an orthogonal transformation which sends l to one of the coordinate axes.)

Questions 7 and 8 are worth 20pts each and the rest 10pts each.